F. Wright

Arthur Hawksley

THE

SCHOLAR's GUIDE

TO

ARITHMETIC;

OR

A COMPLETE EXERCISE-BOOK

FOR THE

USE OF SCHOOLS.

WITH NOTES,

CONTAINING

THE REASON OF EVERY RULE, DEMONSTRATED FROM THE MOST SIMPLE AND EVIDENT PRINCIPLES;

TOGETHER WITH

SOME OF THE MOST USEFUL PROPERTIES OF NUMBERS, AND GENERAL THEOREMS FOR THE MORE EXTENSIVE USE OF THE SCIENCE.

THE SIXTH EDITION.

By JOHN BONNYCASTLE,

OF THE ROYAL MILITARY ACADEMY, WOOLWICH.

LONDON:

PRINTED FOR J. JOHNSON, NO. 72, ST. PAUL'S CHURCH-YARD.

MDCCXCV.

PREFACE

TO THE

SIXTH EDITION.

BOOKS of Arithmetic have of late become ſo extremely numerous, that if the progreſs of the Science were to be eſtimated from that circumſtance alone, it might naturally be concluded that every poſſible improvement had been anticipated, and the ſubject wholly exhauſted. But it has happened in this caſe, as in many others, that much has been promiſed, and little effected. The greater part of theſe performances are ſo nearly alike, both in Matter and Method, that they appear to be little more than mere copies of each other, ill digeſted, and embarraſſed with ſuch a variety of Miſcellaneous obſervations, as render them totally unfit for the purpoſe of teaching.

THE principal object of a work of this kind, ſhould be to provide the learner with a proper ſet of Rules and Examples, ſo methodiſed and arranged, that they may be readily tranſcribed, and fixed in the memory, without any other aſſiſtance from the Maſter, than that of explaining the nature of the proceſs, and examining the truth of the operations. Theſe I have endeavoured to ſupply; and ſince the firſt publication of this Treatiſe, have had the ſatisfaction to find that it has been generally approved by intelligent Tutors, and introduced into ſeve. l of the moſt reſpectable Academies in the kingdom.

To

To render the Work, therefore, ſtill more complete, the preſent Edition has not only been corrected and improved throughout, but in many places entirely re-written.—Every example throughout the book, has, alſo, been ſeparately examined, by two or three different perſons, and the greateſt care taken to avoid errors of the preſs; ſo that it is preſumed few or none will be now found of any material conſequence. To ſay more would be unneceſſary; the plan of the work is already ſufficiently known, and of its merits or defects the public alone muſt determine.

Explanation of the Characters.

$+$ ſignifies plus, or addition.

$-$ ——— minus, or ſubtraction.

$\times$ ——— multiplication.

$\div$ ——— diviſion.

$: \;::\; :$ proportion.

$=$ ——— equality.

$\surd$ ——— ſquare root.

$\sqrt[3]{}$ ——— cube root.

Thus, $4+3$ denotes that 3 is to be added to 4.

$5-2$ denotes that 2 is to be taken from 5.

7×5 denotes that 7 is to be multiplied by 5.

$8\div 4$ denotes that 8 is to be divided by 4.

$2 : 3 :: 4 : 6$ ſhews that 2 is to 3 as 4 is to 6.

$6+4=10$ ſhews that 6 added to 4 is equal to 10.

$\sqrt{2}$, or $2^{\frac{1}{2}}$ denotes the ſquare root of the No 2.

$\sqrt[3]{4}$, or $4^{\frac{1}{3}}$ denotes the cube root of the No 4.

8^2 denotes that the No 8 is to be ſquared.

9^3 denotes that the No 9 is to be cubed, &c.

ARITHMETIC.

ARITHMETIC is the art of computing by Numbers; the rules upon which all its operations depend, being NOTATION, ADDITION, SUBTRACTION, MULTIPLICATION and DIVISION.

NOTATION.

NOTATION teaches to expreſs numbers by words or figures; or to read and write any ſum or number.

The figures by which all numbers may be denoted, are theſe ten, 1 one, 2 two, 3 three, 4 four, 5 five, 6 ſix, 7 ſeven, 8 eight, 9 nine, 0 cypher.

Beſides this value of the figures, they have another, which depends upon the place they ſtand in when joined together; as in the following table.

&c.	Hundreds of Millions.	Tens of Millions.	Millions.	Hundreds of Thousands.	Tens of Thouſands.	Thouſands.	Hundreds.	Tens.	Units.
&c.	9	8	7	6	5	4	3	2	1
		9	8	7	6	5	4	3	2
			9	8	7	6	5	4	3
				9	8	7	6	5	4
					9	8	7	6	5
						9	8	7	6
							9	8	7
								9	8
									9

The figure in the firſt place, reckoning from right to left, denotes only its ſimple value; that in the ſecond place, ten times its ſimple value; that in the third, a hundred times its ſimple value; and ſo on; the value of any figure, in each ſucceſſive place, being always ten times its former value.

Thus, in the number 1786, the 6 in the firſt place ſignifies only ſix; 8 in the ſecond place ſignifies eight tens, or eighty; 7 in the third place, ſeven hundred; the 1 in the fourth place, one thouſand; and the whole number is read thus, one thouſand ſeven hundred and eighty-ſix.

The cypher ſtands for nothing of itſelf, but being joined to the right-hand of other figures, increaſes their value in the ſame ten-fold proportion: thus, 8 ſignifies only eight; but 80 ſignifies eight tens, or eighty; 800 is eight hundred, &c*.

EXAMPLES.

Write in figures the following numbers.

Twenty-five.
One hundred and eighty-nine.
Seven hundred and ſeventeen.
Eight hundred and ſixty.
Nine hundred and five.
One thouſand four hundred and thirty-three.
One hundred and fifty-four thouſand, ſix hundred and fifty.
One million, three hundred thouſand.
One million, two hundred thouſand, ſix hundred and ſeventy five.
Two millions and a half.
Nine hundred and ninety-nine millions, ſeven hundred and ſeventy-ſeven thouſand, five hundred and fifty-five.
Four hundred millions, ſix thouſand and eighty.
Eight hundred and eight millions, eight thouſand, eight hundred and eight.

* For the more eaſily reading of large numbers, they are divided into periods and half-periods, each half-period conſiſting of three figures; the name of the firſt period being units; of the ſecond, millions; of the third, billions; of the fourth, trillions, &c. Alſo the firſt part of any period is ſo many units of it, and the latter part ſo many thouſands.

The following table contains a ſummary of the whole doctrine.

Periods.	Quadrill.	Trillions.	Billions.	Millions.	Units.
Half-per.	th. un.	th. un.	th. un.	th. un.	c.x.t.c.x.u.
Figures.	123,456.	789,098.	765,432.	101,234.	567,890.

Write

Write in words the following numbers:

27	9090	190851
81	10751	940509
170	40848	1207508
1114	85423	8400018
3064	90600	28043713
9876	110101	111000111

SIMPLE ADDITION.

Simple Addition teaches to collect ſeveral numbers of the ſame denomination into one ſum.

Rule*.

1. Place the numbers under each other, ſo that units may ſtand under units, tens under tens, &c. and draw a line under them.

2. Add up the figures in the row of units, and find how many tens are contained in their ſum.

3. Set down what remains above the tens, or, if nothing remains, a cypher, and carry as many ones to the next row as there were tens.

4. Add up the ſecond row, together with the number carried, in the ſame manner as the firſt; and proceed thus till the whole is finiſhed.

Method

---

* This rule, as well as the method of proof, is founded on the known axiom, " the whole is equal to the ſum of all its parts." All that requires explaining, is the method of placing the numbers, and carrying for the tens; both which are evident from the nature of notation; for any other diſpoſition of the numbers would entirely alter their value; and carrying one for every ten, from an inferior line to a ſuperior, is, evidently, right, ſince an unit, in the latter caſe, is equal in value to ten in the former.

Beſides the method here given, there is another very ingenious one of proving addition by caſting out the nines, thus:

Rule 1. Add the figures in the uppermoſt row together, and find how many nines are contained in their ſum.

2. Reject the nines, and ſet down the remainder directly even with the figures in the row.

3. Do the ſame with each of the other rows; and ſet all theſe exceſſes of nine together, in a line, and find their ſum; then, if the exceſs of nines in this ſum, found as before, be equal to the exceſs of nines in the total ſum, the work is right.

METHOD OF PROOF.

1. Draw a line below the uppermost number, and suppose it cut off.

2. Add all the rest together, and set their sum under the number to be proved.

3. Add this last found number and the uppermost line together, and if the sum be the same as that found by the first addition, the work is right.

EXAMPLES.

(1)		(2)		(3)	
23456		22345		34578	
78901		67890		3750	
23456		8752		87	
78901		340		328	
23456		350		17	
78901		78		327	
307071	*Sum*	99755	*Sum*	39087	*Sum*
283615		77410		4509	
307071	*Proof*	99755	*Proof*	39087	*Proof*

4. Add

EXAMPLE.

	Excess of nines.
3782	2
5766	6
8755	7
18303	6

This method depends upon a property of the number 9, which, except 3, belongs to no other digit whatever; viz. that any number divided by 9, will leave the same remainder as the sum of its figures or digits divided by 9; which may be thus demonstrated.

Demon. Let there be any number, as 3467; this, separated into its several parts, becomes $3000 + 400 + 60 + 7$; but $3000 = 3 \times 1000 = 3 \times (999 + 1) = 3 \times 999 + 3$. In like manner $400 = 4 \times 99 + 4$, and $60 = 6 \times 9 + 6$. Therefore $3467 = 3 \times 999 + 4 \times 99 + 6 \times$

4. Add 8635, 2194, 7421, 5063, 2196, and 1245 together. *Ans.* 26754.

5. Add 246034, 298765, 47321, 58653, 64218, 5376, 9821 and 340 together. *Ans.* 730528.

6. Add 562163, 21964, 56321, 18536, 4340, 279 and 83 together. *Ans.* 663686.

7. How many shillings are there in a crown, a guinea, a moidore, and a six and thirty? *Ans.* 89.

8. How many days are there in the twelve calendar months? *Ans.* 365.

9. How many days are there from the 19th day of April 1774, to the 27th day of November 1775, both days exclusive? *Ans.* 586.

SIMPLE SUBTRACTION.

SIMPLE SUBTRACTION teaches to find the difference between any two numbers of the same denomination, by taking the less from the greater.

RULE

$9+3+4+6+7$; and $3467 \div 9 = (3 \times 999 + 4 \times 99 + 6 \times 9 + 3 + 4 + 6 + 7) \div 9$. But $3 \times 999 + 4 \times 99 + 6 \times 9$ is, evidently, divisible by 9; therefore if 3467 be divided by 9, it will leave the same remainder as $3+4+6+7$ divided by 9; and the same will hold for any other number whatever. *Q. E. D.*

The same may be demonstrated universally thus:

Demon. Let $N =$ any number whatever, a, b, c, &c. the digits of which it is composed, and $n =$ as many cyphers as a, the highest digit, is places from unity. Then $N = a$ with n, o's $+ b$ with $(n-1)$ o's $+ c$ with $(n-2)$ o's, &c. by the nature of notation; $= a \times (n-1)$ 9's $+ a + b \times (n-2)$ 9's $+ b + c \times (n-3)$ 9's $+ c$, &c. $= a \times (n-1)$ 9's $+ b \times (n-2)$ 9's $+ c \times (n-3)$ 9's, &c. $+ a + b + c$, &c. but $a \times (n-1)$ 9's $+ b \times (n-2)$ 9's $+ c \times (n-3)$ 9's, &c. is, plainly, divisibly by 9; and therefore N divided by 9 will leave the same remainder as $a + b + c$, &c. divided by 9. *Q. E. D.*

In the same manner this property may be shewn to belong to the number three; but the preference is usually given to the number 9, on account of its being more convenient in practice.

Now, from the demonstration here given, the reason of the rule itself is evident; for the excess of nines in two or more numbers being taken separately, and the excess of nines taken also out of the sum of the former excesses, it is plain this last excess must be equal to the excess of nines contained in the total sum of all these numbers; the parts being equal to the whole.

RULE*.

1. Place the leſs number under the greater, ſo that units may ſtand under units, tens under tens, &c. and draw a line under them.

2. Begin at the right-hand, and take each figure in the lower line from the figure above it, and ſet down the remainder.

3. But if the figure in the lower line be greater than that above it, add ten to the upper one, and then take the lower figure from it.

4. Set down the remainder, and carry one to the next lower figure; with which proceed as before; and ſo on till the whole is finiſhed.

METHOD OF PROOF.

Add the remainder to the leaſt number, and if the ſum be equal to the greateſt, the work is right.

EXAMPLES.

	(1)		(2)		(3)
From	3287625	From	5327467	From	1234567
Take	2343756	Take	1008438	Take	345678
Rem.	943869	*Rem.*	4319029	*Rem.*	888889
Proof.	3287625	*Proof.*	5327467	*Proof.*	1234567

4. From 2637804 Take 2376982. *Anſ.* 260822
5. From 3762162 Take 826541. *Anſ.* 2935621
6. From 78213606 Take 27821890. *Anſ.* 50391716

7. The

This rule was firſt given by Dr. Wallis in his Arithmetic, publiſhed anno 1657, and is a very ſimple eaſy method; though it is liable to this inconvenience, that a wrong operation may ſometimes appear to be right; for, if we change the places of any two figures in the ſum, it will ſtill be the ſame; but then a true ſum will always appear to be ſo, by this proof; and to make a falſe one appear true, there muſt be at leaſt two errors, which are directly oppoſite to each other; and if there be more than two errors, they muſt balance amongſt themſelves: but the chance againſt this particular circumſtance is ſo great, that we may as ſafely truſt to this proof as to any other; except, indeed, when a perſon, who knows the method, has a mind to tranſpoſe the figures in the manner above-mentioned; which muſt always be guarded againſt.

* *Demon.* 1. When all the figures of the leaſt number are leſs than their correſpondent figures in the greater, the difference of the figures

in

7. The Arabian method of notation was first known in England about the year 1150, how long is it since, to this present year 1787? *Ans.* 637 *years.*

8. Sir Isaac Newton was born in the year 1642, and died in 1727, how old was he at the time of his decease? *Ans.* 85 *years.*

9. A grant of the crown, anno domini 1237, was forfeited 137 years before the revolution in 1688; how long did the same subsist? *Ans.* 314 *years.*

10. Homer was born 2520 years ago: How many years was that before the birth of Christ, which is 1787 years ago? *Ans.* 733.

11. Christ was born about the year of the world 4000; the flood happened in the year 1656: How many years was the flood before Christ? *Ans.* 2344.

12. The reformation commenced in the year 1517; King Charles was beheaded in 1648; and his son Charles IId was restored in 1660: How many years were there between each of these events? *Ans.* 131, 12, *and* 143.

13. The mariners compass was invented in 1302; printing in 1440; and America was discovered in 1492: How many years were there between each of these discoveries? *Ans.* 138, 52, *and* 190.

14. Gun-powder was invented in 1344; the Powder Plot was discovered in 1605: How many years were there between, and how many are there since each of these events? *Ans.*

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in the several like places must altogether make the true difference sought; because, as the sum of the parts is equal to the whole, so must the sum of the differences of all the similar parts be equal to the difference of the whole.

2. When any figure of the greater number is less than its correspondent figure in the least, the ten, which is added by the rule, is the value of an unit in the next higher place, by the nature of notation; and as the one which is added to the next place of the less number diminishes the correspondent place of the greater accordingly, this is only taking from one place, and adding as much to another, by which the total is never changed. So that by this means, the greater number is resolved into such parts as are each greater than, or equal to, the similar parts of the less; and therefore the difference of the corresponding figures, taken together, will make up the difference of the whole, as before. *Q. E. D.*

The truth of the method of proof is evident; for the difference of two numbers added to the less is, manifestly, equal to the greater.

MULTI-

MULTIPLICATION TABLE.

2 times	2	is	4
	3		6
	4		8
	5		10
	6		12
	7		14
	8		16
	9		18
	10		20
	11		22
	12		24
3 times	3	is	9
	4		12
	5		15
	6		18
	7		21
	8		24
	9		27
	10		30
	11		33
	12		36
4 times	4	is	16
	5		20
	6		24
	7		28
	8		32
	9		36
	10		40
	11		44
	12		48
5 times	5	is	25
	6		30
	7		35
	8		40
	9		45
	10		50
	11		55
	12		60
6 times	6	is	36
	7		42
	8		48
	9		54
	10		60
	11		66
	12		72
7 times	7	is	49
	8		56
	9		63
	10		70
	11		77
	12		84
8 times	8	is	64
	9		72
	10		80
	11		88
	12		96
9 times	9	is	81
	10		90
	11		99
	12		108
10 times	10	is	100
	11		110
	12		120
11 times	11	is	121
	12		132
12 times	12	is	144

SIMPLE MULTIPLICATION.

SIMPLE MULTIPLICATION is a compendious method of addition, which teaches to find the amount of any given number of one denomination, repeated a certain number of times.

The Number to be multiplied is called the Multiplicand.

The Number you multiply by is called the Multiplier.

The Number found, after the work is finished, is called the Product.

Both the Multiplier and Multiplicand are, in general, called Terms, or Factors.

RULE*.

1. Place the multiplier under the multiplicand, so that units may stand under units, tens under tens, &c. and draw a line under them.

2. Begin at the right-hand, and multiply every figure in the multiplicand by each of the figures in the multiplier.

3. Reckon how many tens there are in the product of every two simple figures, and set down the remainder directly under

* *Demon.* 1. When the multiplier is a single digit, it is plain that the product is properly determined by the rule; for by multiplying every figure by it, that is, every part of the multiplicand, we multiply the whole; and by writing down the products which are less than ten, or the excess of tens, in the places of the figures multiplied, and carrying the number of tens to the product of the next place, is only gathering together the similar parts of the respective products, and is, therefore, the same thing, in effect, as if we wrote down the multiplicand as often as the multiplier expresses, and added them together: for the sum of every column is the product of the figures in the place of that column; and these products collected together are, evidently, equal to the whole required product.

2. If the multiplier be a number made up of more than one digit. After we have found the product of the multiplicand by the first figure of the multiplier, as above, we suppose the multiplier divided into parts, and find, after the same manner, the product of the multiplicand by the second figure of the multiplier; but as the figure we are multiplying by stands

under the figure you are multiplying by, and if nothing remains, a cypher.

4. Carry as many ones as there were tens to the product of the next figures, and proceed, in like manner, till the whole is finiſhed.

5. Add all the products together, and their ſum will be the anſwer required.

METHOD OF PROOF.

Make the former multiplicand the multiplier, and the multiplier the multiplicand; and if the product, found from this operation, is the ſame as before, the work is right.

EXAMPLES.

Mult.	2984	342
by	342	2984
	5968	1368
	11936	2736
	8952	3078
		684
	1020528 *Prod.*	1020528 *Proof.*

3. Multiply

ſtands in the place of tens, the product muſt be ten times its ſimple value; and therefore the firſt figure of this product muſt be placed in the place of tens; or, which is the ſame thing, directly under the figure we are multiplying by. And proceeding in this manner ſeparately with all the figures of the multiplier, it is evident that we ſhall multiply all the parts of the multiplicand by all the parts of the multiplier, or the whole of the multiplicand by the whole of the multiplier; therefore theſe ſeveral products being added together, will be equal to the whole required product. Q. E. D.

The following examples are ſubjoined to make the reaſon of the rule appear as plain as poſſible.

(1)			(2)		
7565			1375435		
5			4567		
25	=	5 × 5	9628045	=	7 times the mult.
300	=	60 × 5	8252610	=	60 times ditto.
2500	=	500 × 5	6877175	=	500 times ditto.
35000	=	7000 × 5	5501740	=	4000 times ditto.
37825	=	7565 × 5	6281611645	=	4567 times ditto.

Beſides

3. Multiply 32745675474 by 2. *Ans.* 65491350948
4. Multiply 37432875643 2 by 3. *Ans.* 112298626929 6
5. Multiply 5806342748 by 4. *Ans.* 23225370992
6. Multiply 84356745674 by 5. *Ans.* 421783728370
7. Multiply 274567546473 by 6. *Ans.* 1647405278838
8. Multiply 54328432847 by 8. *Ans.* 434627462776
9. Multiply 8643597 by 9. *Ans.* 77792373
10. Multiply 796534289 by 11. *Ans.* 8761877179
11. Multiply 3274656461 by 12. *Ans.* 39295877532
12. Multiply 7324687567 by 15. *Ans.* 109870313505
13. Multiply 94713761 by 18. *Ans.* 1704847698
14. Multiply 273580961 by 23. *Ans.* 6292362103
15. Multiply 27501976 by 271. *Ans.* 7453035496
16. Multiply 82164973 by 3027. *Ans.* 248713373271
17. Multiply 6247386495 by 27356. *Ans.* 170903504957220
18. Multiply 8496427 by 874359. *Ans.* 7428927415293
19. Multiply 123456789 by 123456789. *Ans.* 15241578750190521.

CONTRACTIONS.

I. When there are cyphers in the numbers to be multiplied.

RULE.

1. If the cyphers are at the right-hand of the numbers, multiply the other figures only, and place as many cyphers to the right-hand of the product, as are in both the factors.

2. When

Besides the method of proof given above, there is another very convenient and easy one by the help of that peculiar property of the number 9, mentioned in addition; which is performed thus:

RULE 1. Cast the nines out of the two factors, as in addition, and set down the remainders.

2. Multiply the two remainders together, and if the excess of nines in their product be equal to the excess of nines in the total product, the work is right.

EXAMPLE.

4215—3 = excess of 9's in the multiplicand.
878—5 = ditto in the multiplier.

```
   33720
  29505
 33720
 3700770—6 = ditto in the product, = excess of
             9's in 3 × 5.
```

Demon-

2. When the cyphers are in any part of the multiplier, neglect them as before, observing to place the first figure of every product exactly under the figure you are multiplying by.

EXAMPLES.

Mult.	426000	Mult.	8057069
by	22000	by	70050
	852		40285345
	852		56399483
Prod.	9372000000	*Prod.*	564397683450

2. Multiply 461200 by 72000. *Ans.* 33206400000
3. Multiply 815036000 by 70300. *Ans.* 57297030800000

II. When the multiplier is the product of two or more numbers in the table.

RULE*.

Multiply by each of those parts separately, instead of the whole number at once.

Demon. of the Rule. Let M and N be the number of 9's in the factors to be multiplied, and a and b what remains; then $M + a$ and $N + b$ will be the numbers themselves, and their product is $(M \times N) + (M \times b) + (N \times a) + (a \times b)$; but the three first of these products are each a precise number of 9's, because one of their factors is so: these therefore being cast away, there remains only $a \times b$; and if the 9's are also cast out of this, the excess is the excess of 9's in the total product; but a and b are the excesses in the factors themselves, and $a \times b$ their product; therefore the rule is true. *Q. E D.*

This method is liable to the same inconvenience with that in addition.

Multiplication may also, very naturally, be proved by division; for the product being divided by either of the factors will, evidently, give the other; but it would have been contrary to order to have given this rule in the text, because the pupil is supposed, as yet, to be unacquainted with division.

* The reason of this method is obvious; for any number multiplied by the component parts of another, must give the same product as if it were multiplied by that number at once: thus, in example the second, 7 times the given number multiplied by 8, makes 56 times that given number, as plainly as 7 times 8 makes 56.

EXAM-

EXAMPLES.

1. Multiply 123456789 by 25, or by 5 times 5.

$$\begin{array}{r} 123456789 \\ 5 \\ \hline 617283945 \\ 5 \\ \hline 3086419725 \end{array}$$ *the Product.*

2. Multiply 364111 by 56. *Ans.* 20390216
3. Multiply 46123101 by 72. *Ans.* 3320863272
4. Multiply 7128368 by 96. *Ans.* 684323328
5. Multiply 61835720 by 132. *Ans.* 8162315040
6. Multiply 123456789 by 1440. *Ans.* 177777776160

SIMPLE DIVISION.

SIMPLE DIVISION is a compendious method of ſubtraction, which teaches to find how often one number is contained in another of the ſame denomination.

The number to be divided is called the Dividend.

The number you divide by is called the Diviſor.

The number of times the dividend contains the diviſor is called the Quotient.

If the dividend contains the diviſor any number of times, and ſome part or parts over, thoſe parts are called the Remainder.

RULE*.

1. Draw a curved line on the right and left of the dividend, and write the diviſor on the left.

2. Find

* According to the rule, we reſolve the dividend into parts, and find by trial the number of times the diviſor is contained in each of thoſe parts; the only thing then which remains to be proved is, that the ſeveral figures of the quotient, taken as one number, according to the order in which they are placed, is the true quotient of the whole dividend by the diviſor; which may be thus demonſtrated:

Demon. The complete value of the firſt part of the dividend, is, by the nature of notation, 10, 100, or 1000, &c. times the value of which it is taken in the operation, according as there are 1, 2, 3, &c. figures ſtanding before it; and conſequently the true value of the quotient figure belonging to that part of the dividend is alſo 10, 100, or 1000, &c, times its ſimple value.

2. Find how many times the divisor is contained in as many figures of the dividend as are just necessary, and place the number on the right.

3. Multiply the divisor by this number, and place the product under the figures of the dividend above mentioned.

4. Subtract this product from that part of the dividend under which it stands, and bring down the next figure of the dividend, or more if necessary, to the right of the remainder.

5. Divide this number, so increased, as before, and so on till the whole is finished.

N. B. If it be necessary to bring down more figures than one to the remainder, in order to make it larger than the divisor, a cypher must be written in the quotient for every figure so brought down.

METHOD OF PROOF.

Multiply the quotient by the divisor, and if this product, together with the remainder, be equal to the dividend, the work is right.

EXAM-

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value. But the true value of the quotient figure belonging to that part of the dividend, as found by the rule, is also 10, 100. or 1000, &c. times its simple value; for there are as many figures set before it as the number of remaining figures in the dividend; and therefore this first quotient figure taken in its complete value, from the place it stands in, is the true quotient of the divisor in the complete value of the first part of the dividend. For the same reason all the rest of the figures of the quotient taken according to their places, are each the true quotient of the divisor in the complete value of the several parts of the dividend belonging to each; because, as the first figure on the right-hand of each succeeding part of the dividend, has a less number of figures by one standing before it, so in like manner have their quotients: and, consequently taking all the quotient figures in order, as they are placed by the rule, they make one number, which is equal to the sum of the true quotients of all the several parts of the dividend; and this, therefore, is the true quotient of the whole dividend by the divisor. *Q. E. D.*

To leave no obscurity in this demonstration, I shall illustrate it by an example; in which I shall set down the several parts of the dividend and quotient, according to their true values: For this purpose let 8560 be divided by 36, and the work will stand thus:

Divisor

EXAMPLES.

(1)

```
5)135457284565
  ------------
   27091456913 Quot.
  ------------

      338237 Quot.
         365 Divisor.
     -------
     1691185
    2029422
   1014711
   ---------
   123456505 Product.
         284 Rem.
   ---------
   123456789 Proof.
```

(2)

```
365)123456789(338237 Quot.
    1095
    ----
     1395
     1095
     ----
      3006
      2920
      ----
       867
       730
       ---
       1378
       1095
       ----
        2839
        2555
        ----
         284 Rem.
```

3. Divide

	Divisor 36)8560 dividend.		
1st. part of the dividend.	8500		
36 × 200 =	7200	......	200 the 1st. quotient.
1st. remainder	1300		
add ..	60		
2d. part of the dividend.	1360		
36 × 30 =	1080	......	30 the 2d. quotient.
2d. remainder	280		
add ..	0		
3d. part of the dividend ..	280		
36 × 7 =	252	......	7 the 3d. quotient.
Last remainder	28	......	237 sum of all the quotients, or answer.

When there is no remainder to a division, the quotient is the absolute and perfect answer to the question; but where there is a remainder, it may be observed, that it goes so much towards another time as it ap-

3. Divide 3756789275474 by 2. *Ans.* 1878394637737
4. Divide 5474857647651 by 3. *Ans.* 1824952549217
5. Divide 653783754732 by 4. *Ans.* 163445938683
6. Divide 2345678964 by 6. *Ans.* 390946494
7. Divide 12345678900 by 7. *Ans.* $1763668414\frac{2}{7}$
8. Divide 9876543210 by 8. *Ans.* $1234567901\frac{2}{8}$
9. Divide 1357975313 by 9. *Ans.* $150886145\frac{8}{9}$
10. Divide 570196382 by 12. *Ans.* $47516365\frac{2}{12}$
11. Divide 3217684329765 by 17. *Ans.* $189275548809\frac{12}{17}$
12. Divide 321147368 by 27. *Ans.* $11894346\frac{26}{27}$
13. Divide 137896254 by 97. *Ans.* $1421610\frac{84}{97}$
14. Divide 1406373 by 108. *Ans.* $13021\frac{105}{108}$
15. Divide 3405657254 by 345. *Ans.* $9871470\frac{104}{345}$
16. Divide 5713070046 by 678. *Ans.* 8426357
17. Divide 293839455936 by 8405. *Ans.* $34960078\frac{7246}{8405}$
18. Divide 4637064283 by 57606. *Ans.* $80496\frac{11707}{57606}$
19. Divide 352107193214 by 210472. *Ans.* $1672940\frac{165534}{210472}$
20. Divide 55800117726061767724 by 2708630425.
Ans. 206008604—24 *rem.*

CON-

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proaches to the divisor; thus, if the remainder be a fourth part of the divisor, it will go one fourth of a time more; if half the divisor, it will go the half of a time more; and so on. In order, therefore, to complete the quotient, put the last remainder at the end of it, above a small line, and the divisor below it.

As it is sometimes difficult to find how often the divisor may be had in the numbers of the several steps of the operation; the best way will be to find how often the first figure of the divisor may be had in the first, or two first, figures of the dividend, and the answer made less by one or two is generally the figure wanted: besides, if after subtracting the product of the divisor and quotient from the dividend, the remainder be equal to, or exceed the divisor, the quotient figure must be increased accordingly.

The reason of the method of proof is plain; for since the quotient is the number of times the dividend contains the divisor, the product of the quotient and divisor must, evidently, be equal to the dividend.

There are several other methods made use of to prove division, the best and most useful of which are the following.

Rule I. Subtract the remainder from the dividend, and divide this number by the quotient, and the quotient found by this division will be equal to the former divisor, when the work is right.

Rule II. Add the remainder, and all the products of the several quotient figures by the divisor together according to the order in which they stand in the work, and the sum will be equal to the dividend, when the work is right.

Rule

CONTRACTIONS.

I. When cyphers are annexed to the divisor.

RULE*.

Cut off the cyphers from the divisor, and the same number of figures from the right-hand of the dividend; then divide the remaining figures by each other, as usual, and the quotient will be the answer.

If any thing remains after this division, place the figures cut off from the dividend to the right-hand of it, and it will be the true remainder.

EXAMPLES.

1. Divide 46748696 by 20

```
2,0)4674869,6
    ---------
    2337434—16/20 Quotient.
```

Rule III. Subtract the remainder from the dividend, and what remains will be equal to the product of the divisor and quotient; which may be proved by casting out the nines as was done in multiplication.

To avoid obscurity, I shall give an example proved according to all the different methods.

```
87)12689(145 Quot.          12689
  .87..                        74
  ----                     ------
   398                 145)12615(87 Proof by Division.
  .348.                    1160
  ----                     ----
   509                     1015
  ..435                    1015
  ----                     ----
  ...74
  -----
  12689                     145
                             87
Proof by casting out the 9's. ----
  1 = excess of 9's in 145   1015
  6 = ditto in 87           1160
  --                          74
1×6=6= ditto in 12689—74.  -----
                           12689 Proof by Multiplication.
```

* The reason of this contraction is easy to conceive: for the cutting off the same numbers of figures from each, is the same as dividing each of them by 10, 100, 1000, &c. and it is evident, that as often as the whole divisor is contained in the whole dividend, so often must any part of the divisor be contained in a like part of the dividend:—This method is only to avoid a needless repetition of cyphers, which would happen in the common way, as may be seen by working an example at large.

2. Divide 310869017 by 7100.

```
71,00)3108690,17(43784 2617/7100 Quotient.
      284
      ---
      268
      213
      ---
       556
       497
       ---
        599
        568
        ---
         310
         284
         ---
         2617
```

3. Divide 7380964 by 23000. *Ans.* $320\frac{20964}{23000}$

4. Divide 29628754963 by 35000. *Ans.* $846535\frac{29963}{35000}$

II. When the divisor is the product of two or more numbers in the table.

RULE *.

Divide by each of those numbers separately, instead of the whole divisor at once.

EXAM-

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* This follows from contraction the 3d. in multiplication, of which it is only the converse; for the third part of the half of any thing is, evidently, the same as the sixth part of the whole; and so of any other number.

The true remainder, in questions wrought by this contraction, is found as follows:

Rule. Multiply the quotient by the divisor, and subtract the product from the dividend, and the result will be the true remainder.

The truth of this is extremely obvious; for if the product of the divisor and quotient, added to the remainder, be equal to the dividend, their product taken from the dividend must leave the remainder.

But the rule which is most commonly made use of is this:

Rule. Multiply the last remainder by the preceding divisor, or last but one, and to the product add the preceding remainder; multiply this sum by the next preceding divisor, and to the product add the next preceding remainder; and so on, till you have gone through all the divisors and remainders to the first.

EXAM-

EXAMPLES.

1. Divide 31046835 by 56.

$$\begin{array}{r} 7)31046835 \\ \hline 8)4435262-1 \\ \hline 554407-6 \end{array}$$ *the quotient.*

2. Divide 7014596 by 72. *Ans.* $97424\frac{68}{72}$
3. Divide 5130652 by 132. *Ans.* $38868\frac{76}{132}$
4. Divide 83016572 by 240. *Ans.* $345902\frac{92}{240}$

III. To perform division more concisely than by the general rule.

RULE*.

Multiply the divisor by the quotient figures as before, and subtract each figure of the product from the dividend, as you produce it; always remembering to carry as many to the next figure as were borrowed before.

EXAMPLE.

Let 64865 be divided by 144.

$$\begin{array}{r} 9)64865 \\ \hline 4)7207-2 \\ \hline 4)1801-3 \\ \hline 450-1 \end{array}$$

Ans. $450\frac{65}{144}$

	1	the last remainder.
Mult.	4	the preceding divisor.
	4	
Add	3	the 2d. remainder.
	7	
Mult.	9	the first divisor.
	63	
Add	2	the first remainder.
	65	

To explain this rule from the example, we may observe, that every unit of the 1st quotient may be looked upon as containing 9 of the units in the given dividend; consequently every unit that remains will contain the same; therefore this remainder must be multiplied by 9 in order to find the units it contains of the given dividend. Again, every unit in the next quotient will contain 4 of the preceding ones, or 36 of the first, that is, 9 times 4; therefore what remains must be multiplied by 36; or, which is the same thing, by 9 and 4 continually. Now, this is the same as the rule; for instead of finding the remainders separately, they are reduced from the bottom upwards, step by step, to one another, and the remaining units of the same class taken in as they occur.

* The reason of this rule is the same as that of the general rule, p. 15.

EXAMPLES.

1. Divide 3104675846 by 833.

$$
\begin{array}{l}
833)3104675846(3727101\tfrac{8}{833}\ \textit{the quotient.} \\
\quad\ 6056 \\
\quad\quad 2257 \\
\quad\quad\ 5915 \\
\quad\quad\quad 848 \\
\quad\quad\quad\ 1546 \\
\quad\quad\quad\quad 713
\end{array}
$$

2. Divide 29137062 by 5317. *Ans.* $5479\frac{5219}{5317}$
3. Divide 62015735 by 7803. *Ans.* $7947\frac{5294}{7803}$
4. Divide 432756284563574 by 873469. *Ans.* $495445498\frac{872012}{873469}$.

COMPOUND ADDITION.

COMPOUND ADDITION teaches to collect several numbers of different denominations into one sum.

RULE*.

1. Place the numbers so that those of the same denomination may stand directly under each other, and draw a line below them.
2. Add up the figures in the lowest denomination, and find how many units, or ones, of the next higher denomination are contained in their sum.
3. Write down the remainder, and carry the ones to the next denomination, which add up in the same manner as before.
4. Proceed thus through all the denominations to the highest, whose sum, together with the several remainders, will give the answer required.

The method of proof is the same as in simple addition.

---

* The reason of this rule is evident from what has been said in simple addition: for, in addition of money, as 1 in the pence is equal to 4 in the farthings; 1 in the shillings to 12 in the pence; and 1 in the pounds to twenty in the shillings; therefore, carrying as directed, is nothing more than providing a method of placing the money arising from each column properly in the scale of denominations; and this reasoning will hold good in the addition of compound numbers of any denomination whatsoever.

TABLES

TABLES OF MONEY.

2 Farthings	make	1 Halfpenny	½	qrs.	d.			
4 Farthings	——	1 Penny	d.	4=	1	s.		
12 Pence	——	1 Shilling	s.	48=	12=	1	£.	
20 Shillings	——	1 Pound	£.	960=	240=	20=	1	

PENCE TABLES.

d.		s.	d.		s.	d.
12	make	1	20	is	1	8
24	——	2	30	—	2	6
36	——	3	40	—	3	4
48	——	4	50	—	4	2
60	——	5	60	—	5	0
72	——	6	70	—	5	10
84	——	7	80	—	6	8
96	——	8	90	—	7	6
108	——	9	100	—	8	4
120	——	10	110	—	9	2
			120	—	10	0

EXAMPLES.

	l.	s.	d.	l.	s.	d.	l.	s.	d.
	173	13	5	705	17	3½	1275	12	4
	87	17	7¾	354	17	2¾	700	10	10½
	75	18	7½	175	17	3¾	25	13	3¾
	25	17	8¼	87	19	7½	5	17	7¾
	10	10	10½	52	12	7¾	0	18	8
	2	5	7	27	10	5¼	0	17	0
Sum	376	3	10	1404	14	6½	2009	9	10
	202	10	5	698	17	3	733	17	6
Proof	376	3	10	1404	14	6½	2009	9	10

	l.	s.	d.	l.	s.	d.	l.	s.	d.
	228	14	6	678	13	6¼	678	5	10
	327	18	4½	287	6	2	87	10	9¼
	579	12	6¾	438	15	0¼	123	8	8
	109	18	10	325	17	2	47	16	9
	730	10	1½	840	12	9¼	307	2	0
	185	14	2	426	17	8½	187	16	10¼
Sum									

	l.	s.	d.	l.	s.	d.	l.	s.	d.
	368	10	3	567	8	9	1728	10	$8\frac{1}{2}$
	257	10	5	259	16	$8\frac{3}{4}$	457	10	6
	88	11	$4\frac{1}{2}$	287	6	$7\frac{3}{4}$	328	19	$9\frac{3}{4}$
	33	10	0	87	15	4	478	12	$2\frac{1}{2}$
	12	13	5	25	16	8	238	14	10
	8	8	$8\frac{1}{2}$	24	10	2	50	10	$6\frac{1}{4}$
Sum									
Proof									

A. owes B. for bread, 9 *l.* 6 *s.* $3\frac{1}{4}$ *d.*; for cheese, 4 *l.* 3 *s.*; for tea, 10 *l.* 9 *s.* 5 *d.*; for butter, 3 *l.* $2\frac{1}{4}$ *d.*; for sugar, 125 *l.* $\frac{3}{4}$ *d.*; for other articles, 26 *l.* 13 *s.* $6\frac{3}{4}$ *d.* What is the amount of the whole debt?

EXAMPLES OF WEIGHTS AND MEASURES.

TROY WEIGHT.

TABLES.

Grains.			gr.	gr.	dwt.		
24 Grains make	1 Pennyweight	*dwt.*		24 =	1	*oz.*	
20 Pennyweights	1 Ounce	*oz.*		480 =	20 =	1	*lb.*
12 Ounces ——	1 Pound	*lb.*		5760 =	240 =	12 =	1

By this Weight are weighed Gold, Silver, Jewels, and Liquors.

EXAMPLES.

	lb.	oz.	dwt.	gr.		lb.	oz.	dwt.	gr.
Add	14	6	12	13	Add	10	8	11	17
	17	5	3	12		42	5	16	12
	15	0	9	16		12	2	14	18
	2	7	15	20		51	6	0	22
	13	2	10	19		24	9	17	17
	4	1	5	21		29	4	18	22
Sum	66	11	18	5	Sum	171	3	0	12

	lb.	oz.	dwt.	gr.
Add	100	10	19	20
	432	6	0	5
	80	3	2	1
	7	0	0	9
	0	11	19	23
	0	0	8	9
Sum	621	8	10	19

	lb.	oz.	dwt.	gr.
Add	171	6	13	14
	391	11	9	12
	230	6	6	13
	94	7	3	18
	42	10	15	20
	31	0	0	21

	lb.	oz.	dwt.	gr.
Add	49	8	7	10
	56	3	13	23
	99	11	19	1
	9	9	9	0
	0	10	3	2
	0	0	18	20

	lb.	oz.	dwt.	gr.
Add	27	10	17	18
	17	9	12	14
	33	6	13	15
	0	11	13	15
	0	0	19	8
	0	0	0	23

What is the sum of 48 *lb.* 11 *oz.* 18 *dwt.* 21 *gr.*; 42 *lb.* 10 *oz.* 14 *dwt.*; 40 *lb.* 9 *oz.* 16 *dwt.* 20 *gr.*; 36 *lb.* 8 *oz.* 15 *dwt.* 22 *gr.*; 38 *lb.* 10 *oz.* 10 *dwt.*; 53 *lb.* 17 *dwt.* 13 *gr.*?

Anſ. 261 *lb.* 4 *oz.* 13 *dwts.* 4 *gr.*

APOTHECARIES WEIGHT.

TABLES.

Grains.			gr.		
20 Grains	make	1 Scruple	*ſc.*	or	℈
3 Scruples	——	1 Dram	*dr.*	or	ʒ
8 Drams	——	1 Ounce	*oz.*	or	℥
12 Ounces	——	1 Pound	*lb.*	or	℔

Grains.

20 = 1 Scruple.

60 = 3 = 1 Dram.

480 = 24 = 8 = 1 Ounce.

5760 = 288 = 96 = 12 = 1 Pound.

Apothecaries uſe this weight in compounding their medicines, but buy and ſell their drugs by Avoirdupois weight.

EXAM-

EXAMPLES.

℔	℥	ʒ	℈	gr.
24	7	2	1	16
17	11	7	2	19
36	6	5	0	7
15	9	7	1	13
9	3	4	1	9
16	10	3	2	17
4	0	1	1	12
125	2	1	0	13

℥	ʒ	℈	gr.
11	2	1	17
7	4	2	14
4	0	1	19
2	5	2	11
10	1	2	16
8	7	1	13
9	0	0	11
53	7	2	1

ʒ	℈	gr.
3	2	15
0	1	13
2	2	11
7	0	17
5	2	14
6	1	0
0	0	19
26	0	9

lb.	*oz.*	*dr.*	*ſc.*	*gr.*
17	6	5	2	18
33	9	1	0	4
20	8	7	1	11
86	11	3	2	9
100	4	0	0	19
232	10	6	2	1

oz.	*dr.*	*ſc.*	*gr.*
8	5	1	8
7	6	2	13
11	7	0	0
10	0	0	16
1	2	2	3
0	7	1	19

dr.	*ſc.*	*gr.*
7	1	17
4	0	3
0	2	10
4	1	12
6	0	0
7	2	19

An apothecary made a compoſition of five ingredients; the firſt weighed 3 *lb.* 7 *oz.* the ſecond, 11 *oz.* 7 *dr.* 13 *gr.* the third, 7 *lb.* 2 *ſc.* the fourth, 1 *lb.* 3 *dr.* 1 *ſc.* and the fifth, 5 *lb.* 5 *oz.* 2 *dr.* 1 *ſc.* 7 *grs.* What was the weight of the whole?

AVOIRDUPOIS WEIGHT.

TABLES.

Drams.			*dr.*
16 Drams	make	1 Ounce	*oz.*
16 Ounces	——	1 Pound	*lb.*
28 Pounds	——	1 Quarter	*qrs.*
4 Quarters	——	1 Hundred Weight	*cwt.*
20 Hundred Weight	——	1 Ton	*Ton.*

Drams.

16 = 1 Ounce.
256 = 16 = 1 Pound.
7168 = 448 = 28 = 1 Quarter.
28672 = 1792 = 112 = 4 = 1 Hund. Wt.
573440 = 35840 = 2240 = 80 = 20 = 1 Ton.

By this Weight are weighed all Things of a coarse or drossy nature, as Butter, Cheese, Flesh, Grocery Wares, and some Liquids, Bread, Corn, &c. and all Metals except Gold and Silver: But several kinds of Silks are weighed by a *lb.* of 24 *oz.* and Butter, in some Countries, is from 16 to 32 *oz.* in a *lb.*

EXAMPLES.

T.	*Cwt.*	*qr.*	*lb.*	*oz.*
42	14	2	20	14
59	12	1	14	7
76	13	3	22	12
47	17	1	17	4
36	10	2	9	10
49	9	1	16	9
57	14	2	8	6
3	4	3	24	13
373	17	3	22	11

	Cwt.	*qr.*	*lb.*	*oz.*	*dr.*
	20	3	27	15	13
	12	0	0	6	2
	10	1	3	9	4
	6	2	8	1	15
	4	2	20	13	3
	27	0	21	2	6
	8	1	2	0	0
T.	0	2	13	12	11
4	10	2	13	13	6

Cwt.	*qr.*	*lb.*	*oz.*	*dr.*
14	1	25	14	9
13	2	20	1	15
9	3	6	7	3
10	0	18	12	11
7	2	27	3	2
6	1	19	8	1
4	3	0	15	5
12	2	0	0	13

T.	*Cwt.*	*qr.*	*lb.*	*oz.*
15	12	1	10	10
71	8	2	6	0
83	19	3	15	5
36	7	0	20	14
47	11	1	27	11
63	5	2	19	7
12	13	1	14	9
9	7	0	5	10

Cwt.	*qr.*	*lb.*	*oz.*	*dr.*
24	3	25	10	8
17	2	1	3	11
8	1	2	14	0
9	0	26	12	15
4	1	0	5	1

Cwt.	*qr.*	*lb.*	*oz.*
51	3	19	0
17	0	26	15
18	1	12	8
12	2	0	14
4	1	0	10

A Shopkeeper buys 3 *qrs.* 14 *lb.* of Teas; 1 *qr.* 23 *lb.* of Coffee; 3 *Cwt.* 2 *qrs.* 5 *lb.* of Sugars; 2 *qrs.* 3 *lb.* 13 *oz.* 9 *dr.* of Spices; 13 *Cwt.* 1 *qr.* 24 *lb.* of Hops, and other

Articles weighing 3 *Cwt.* 17 *lb.* 7 *oz.* 13 *dr.* What is the Weight of the Whole?

Ans. 22 *Cwt.* 3 *lb.* 5 *oz.* 6 *dr.*

WOOL WEIGHT.

TABLES.

7 Pounds	make	1 Clove	-	*Cl.*
2 Cloves	——	1 Stone		*St.*
2 Stone	——	1 Tod		*T.*
6 Tods and a Half	——	1 Wey		*W.*
2 Weys	——	1 Sack		*Sa.*
12 Sacks	——	1 Last		*La.*

And a Pack is 12 Score *lb.*

Pounds.

7 = 1 Clove.
14 = 2 = 1 Stone.
28 = 4 = 2 = 1 Tod.
182 = 26 = 13 = 6½ = 1 Wey.
364 = 52 = 26 = 13 = 2 = 1 Sack.
4368 = 624 = 312 = 156 = 24 = 12 = 1 Last.

EXAMPLES.

La.	*Sa.*	*W.*	*T.*	*St.*	*Cl.*
21	9	1	5	1	1
18	7	0	4	1	0
9	10	1	6	0	1
7	11	1	3	1	1
8	1	0	2	1	1
66	5	0	3½	0	0

W.	*T.*	*S.*	*Cl.*	*lb.*
1	4	1	1	6
0	2	0	1	4
1	6	1	0	3
0	5	0	1	1
0	4	1	1	6
5	4½	0	0	6

La.	*Sa.*	*W.*	*T.*	*St.*	*Cl.*
29	8	1	4	1	1
19	7	1	6	1	1
8	6	0	5	0	0
0	10	1	3	1	1
0	0	0	6	1	0
34	9	1	2	0	1

T.	*St.*	*Cl.*	*lb.*
12	1	1	3
14	0	1	4
15	1	0	2
13	1	1	6
9	1	0	5
0	1	1	3

Sa.	W.	T.	St.	Cl.	lb.
45	1	3	1	0	6
17	0	6	0	1	5
28	1	0	1	1	4
13	0	5	1	0	3
9	1	4	1	1	2

W.	T.	St.	Cl.	lb.
85	4	1	1	6
73	2	1	0	5
69	5	0	1	3
42	1	1	1	4
38	6	1	1	2

LONG MEASURE.

TABLES.

	Barley Corns.			*Bar.*
3	Barley Corns	make	1 Inch	*In.*
12	Inches	——	1 Foot	*Ft.*
3	Feet	——	1 Yard	*Yd.*
6	Feet	——	1 Fathom	*Fth.*
5	Yards and a Half	——	1 Pole or Rod	*P.*
40	Poles	——	1 Furlong	*Fur.*
8	Furlongs	——	1 Mile	*Mile.*
3	Miles	——	1 League	*Lea.*
60	Miles, or 69½	——	1 Degree	*Deg.* or °

Barley Corns.

3 =	1 Inch.					
36 =	12 =	1 Foot.				
108 =	36 =	3 =	1 Yard.			
594 =	198 =	16½ =	5½ =	1 Pole.		
23760 =	7920 =	660 =	220 =	40 =	1 Furlong.	
190080 =	63360 =	5280 =	1760 =	320 =	8 =	1 Mile.

EXAMPLES.

Lea.	Mil.	Fur.	P.
20	2	7	38
18	1	5	20
16	0	4	39
25	2	0	6
9	1	2	0
8	2	1	25
99	1	6	8

P.	Yds.	Ft.	In.	Bar.
20	4	2	11	1
10	1	1	8	2
13	2	0	7	1
31	0	1	10	2
12	5	2	0	1
5	3	1	6	0
94	1½	1	8	1

Mil.	*Fur.*	*P.*	*Yds.*	*Ft.*	*In.*
37	3	14	2	1	5
28	4	17	3	2	10
17	4	4	3	1	2
10	5	6	3	1	7
29	2	2	2	0	3
30	0	0	4	0	2

Lea.	*Mil.*	*Fur.*	*P.*	*Yds.*
13	1	7	10	4
40	2	6	30	3
15	1	0	12	2
29	0	7	29	0
64	1	0	17	1
98	2	5	0	5

Fur.	*P.*	*Yds.*	*Ft.*	*In.*	*Bar.*
6	35	4	2	11	1
1	12	2	1	8	2
9	16	5	2	0	1
7	24	0	1	9	0
8	38	3	2	10	1
9	0	0	1	6	2

Mil.	*Fur.*	*P.*	*Yds.*	*Ft.*
156	7	19	4	2
213	3	36	5	1
701	0	10	2	0
91	6	25	0	1
0	4	13	3	2
0	0	38	2	0

From A to B is 3 *Mil.* 2 Fur. 7 *P.*; from B to C 17 *Mil.* 13 *P.*; from C to D 7 *Fur.* 10 *P.* 5 *Yds.*; and from D to E 5 *Mil.* 33 *P.* 1 *Yd.* 7 *In.* What is the Distance from A to E?

Ans. 26 *Mil.* 2 *Fur.* 24 *Pls.* 2 *Ft.* 1 *In.*

CLOTH MEASURE.

TABLES.

2 Inches and a Quarter	make	1 Nail	*Nl.*
4 Nails	——	1 Quarter of Yd.	*Qrs.*
3 Quarters	——	1 Flemish Ell	*F. E.*
4 Quarters	——	1 Yard	*Yd.*
5 Quarters	——	1 English Ell	*E. E.*
6 Quarters	——	1 French Ell	*Fr. E.*

Inches.

$2\frac{1}{4} =$ 1 Nail.

9 = 4 = 1 Quarter.

36 = 16 = 4 = 1 Yard.

27 = 12 = 3 = 1 Flemish Ell.

45 = 20 = 5 = 1 English Ell.

54 = 24 = 6 = 1 French Ell.

EXAM-

EXAMPLES.

F.E.	Qrs.	N.	In.	Yds.	Qrs.	N.	In.	E.E.	Qrs.	Ns.	In.
65	1	3	1	20	3	1	1	97	2	2	1
26	2	1	2	38	2	0	1	58	1	3	2
24	0	1	0	28	2	0	2	20	4	2	1
82	2	3	1	45	1	3	1	9	3	0	2
33	0	3	0	63	0	2	0	0	4	3	1
7	1	2	1	8	2	1	2	0	2	2	2
240	0	3	$\frac{1}{2}$	205	0	2	$\frac{1}{4}$	188	0	0	0

Fr.El.	Qrs.	Nl.	Yds.	Qrs.	N.	I.	En.El.	Qr.	Nls.
126	4	2	785	2	3	1	950	3	3
233	5	3	392	3	2	2	837	4	2
87	1	2	86	1	1	0	903	2	1
32	3	1	7	0	2	1	250	1	0
25	2	0	0	3	1	2	501	0	3
16	0	2	0	0	2	1	69	3	2

A Merchant bought 4 Parcels of Cloth; the first contained 4[illegible]0 *En. Ells* 1 *Ya.* 3 *Nls.*; the second 976 *Ells* 3 *Qrs.*; the third 765 *Yds.* 2 *Qrs.* 1 *Nl.*; the fourth 43 *Ells* 1 *Yd.* How many Ells, &c. were there in the whole?

SQUARE MEASURE.

TABLES.

144	Inches	make	1	Square Foot	*Ft.*
9	Square Feet	——	1	Square Yard	*Yd.*
$30\frac{1}{4}$	Square Yards	——	1	Square Pole	*P.*
40	Square Poles	——	1	Rood,	*Rd.*
4	Roods	——	1	Acre	*Acr.*

Inches.

144 = 1 Foot.
1296 = 9 = 1 Yard.
39204 = $272\frac{1}{4}$ = $30\frac{1}{4}$ = 1 Pole.
1568160 = 10890 = 1210 = 40 = 1 Rood.
6272640 = 43560 = 4840 = 160 = 4 = 1 Acre.

By this Measure, Land, Husbandmen and Gardeners Work are measured; and Board, Glass, Pavements, Plaistering, Wainscot-

Wainſcotting, Tiling, Flooring, and every Dimenſion of Length and Breadth only.

When Length, Breadth and Depth are taken into conſideration, it is called Solid or Cubic Meaſure, which is uſed to meaſure Timber, Stone, &c.

The ſolid Foot, which is 12 Inches in Length, Breadth, and Depth, contains 1728 Inches; and 27 ſolid Feet are a ſolid Yard.

EXAMPLES.

Rd.	*Pl.*	*Yds.*	*Ft.*		*Acr.*	*Rd.*	*Pl.*		*Acr.*	*R.*	*Pl.*
3	38	26	7		382	1	34		721	2	15
2	15	13	5		618	3	14		94	3	32
1	2	6	2		100	1	27		36	2	29
0	1	9	4		74	2	19		59	3	28
2	0	0	3		63	1	31		265	0	17
3	20	30	8		55	3	38		27	0	30
12	38	26¾	2		1295	3	3		1205	1	31

Rd.	*Pl.*	*Yds.*	*Ft.*		*Acr.*	*Rd.*	*Pl.*		*Acr.*	*Rd.*	*Pl.*
2	1	28	6		409	1	36		4061	0	24
3	30	10	7		81	3	20		2731	2	3
1	38	30	3		94	2	10		841	3	19
0	18	0	2		8	0	17		96	2	39
1	0	12	0		0	3	39		85	0	10
1	20	13	3		0	0	25		40	1	0

A Surveyor having meaſured 4 Pieces of Land, found one to contain 7 Acres, 3 Roods, 24 Poles; another, 18 Acres, 1 Rood, 16 Poles; the third, 20 Acres, 5 Poles, 8 Yards; and the fourth, 15 Acres, 24 Yards, 7 Feet. How many Acres, &c. were ſurveyed?

WINE MEASURE.

TABLES.

	Pints			*Pt.*
2	Pints	make	1 Quart	*Qts.*
4	Quarts	——	1 Gallon	*Gal.*
42	Gallons	——	1 Tierce	*Tier.*
2	Tierces	——	1 Puncheon	*Pun.*
63	Gallons	——	1 Hogſhead	*Hhd.*
2	Hogſheads	——	1 Pipe or Butt	*P.*
2	Pipes	——	1 Tun	*T.*

Pints.

Pints.							
2 =	1 Quart.						
8 =	4 =	1 Gallon.					
336 =	168 =	42 =	1 Tierce.				
504 =	252 =	63 =	$1\frac{1}{2}$ =	1 Hogshead.			
672 =	336 =	84 =	2 =	$1\frac{1}{3}$ =	1 Puncheon.		
1008 =	504 =	126 =	3 =	2 =	$1\frac{1}{2}$ =	1 Pipe.	
2016 =	1008 =	252 =	6 =	4 =	3 =	2 =	1 Tun.

By this Measure, Brandies, Spirits, Cyder, Mead, Vinegar, Oil, Honey, &c. are measured.—A gallon contains 231 cubic inches.

EXAMPLES.

P.	*Hhd.*	*Gal.*	*Qts.*	*Pts.*
1	1	27	3	1
0	1	60	1	0
1	0	34	0	1
0	1	37	2	1
1	1	52	1	1
1	0	48	3	1
1	0	42	2	1
9	0	51	3	0

Hhd.	*Gal.*	*Qt.*	*Pt.*
64	22	2	1
21	17	3	1
73	61	2	1
63	45	1	1
40	20	3	1
27	16	2	0
94	50	3	1
385	46	3	0

T.	*P.*	*Hhd.*	*Gal*	*Qt.*
83	1	1	62	3
32	0	0	12	1
80	1	1	40	2
91	1	0	20	0
53	1	1	55	3
42	1	0	0	2
9	0	1	10	1

Tun.	*Pun.*	*Tier.*	*Gal.*
61	2	1	40
53	1	0	39
48	2	1	13
32	0	0	10
25	1	1	9
17	2	0	41
8	1	1	0

Tun.	*Pun*	*Tier.*	*Gal.*	*Qt.*
56	2	0	41	3
32	1	1	16	2
48	2	1	10	1
25	0	0	38	0
10	2	1	19	2
8	0	1	0	3
0	2	0	40	1

Hhd.	*Gal.*	*Qt.*	*Pt.*
53	12	2	1
91	61	3	1
81	0	2	1
90	15	0	0
8	6	2	1
0	57	1	0
0	0	3	1

A Merchant imported 8 Tuns of Claret; 12 Tuns, 1 Hogshead, 9 Gallons of Port; 4 Tuns, 1 Pipe, 1 Puncheon of Sherry; 3 Hogsheads, 12 Gallons, 3 Quarts of Lisbon. How many Tuns, &c. were imported in the Whole?

ALE AND BEER MEASURE.

TABLES.

2 Pints	make	1 Quart	*Qt.*
4 Quarts	——	1 Gallon	*Gal.*
8 Gallons	——	1 Firkin of Ale	*A. Fir.*
9 Gallons	——	1 Firkin of Beer	*B. Fir.*
2 Firkins	——	1 Kilderkin	*Kil.*
2 Kilderkins	——	1 Barrel	*Bar.*
1 Barrel and a Half	——	1 Hogshead	*Hhd.*
2 Barrels	——	1 Puncheon	*Pun.*
2 Hogsheads	——	1 Butt	*Butt*
2 Butts	——	1 Tun	*Tun*

Pints.

2 = 1 Quart.

8 = 4 = 1 Gallon.

72 = 36 = 9 = 1 Firkin.

144 = 72 = 18 = 2 = 1 Kilderkin.

288 = 144 = 36 = 4 = 2 = 1 Barrel.

432 = 216 = 54 = 6 = 3 = $1\frac{1}{2}$ = 1 Hogshead.

576 = 288 = 72 = 8 = 4 = 2 = $1\frac{1}{3}$ = 1 Puncheon.

864 = 432 = 108 = 12 = 6 = 3 = 2 = $1\frac{1}{2}$ = 1 Butt.

Note. A gallon of ale contains 282 cubic inches.

EXAMPLES.

Butts.	*Hhds.*	*Bar.*	*Kil.*	*B.Fir.*	*Gal.*
56	1	1	1	1	8
19	0	1	1	1	7
30	1	0	1	1	6
47	1	1	0	1	5
25	1	1	1	0	3
15	1	1	1	1	0
197	1	0	1	0	2

Hhds.	*B.Fir.*	*Gal.*	*Qt.*	*Pt.*
45	2	7	3	1
36	3	6	2	1
95	1	5	1	0
86	1	4	3	1
17	3	4	0	1
10	0	2	3	1
291	1	4	2	1

Tuns

Tuns	But.	Hhds.	Gal.	Qt.
32	1	1	27	3
23	0	1	51	2
98	1	0	39	1
46	1	1	12	0
12	0	1	9	4
56	1	0	28	2

Hhds.	Gal.	Qt.	Pt.
90	50	2	1
19	35	3	0
78	16	1	1
16	3	0	1
9	52	3	0
8	13	2	1

Pun.	A.Fr.	Gal.	Qt.	Pt.
365	7	6	3	1
84	5	7	2	0
10	2	3	1	1
0	6	2	3	1
0	0	5	1	0

Hhds.	Kil.	Gal.	Qt.	Pt.
98	2	17	3	1
54	1	16	2	0
33	0	10	1	1
20	1	8	0	1
11	2	6	3	1

A Brewer ſent to an Inn-keeper at one Time 5 Hogſheads, 1 Barrel, 20 Gallons of Beer; at another, 9 Kilderkins, 1 Firkin; and at another, 1 Tun, 3 Hogſheads, 50 Gallons. How many Tuns, Hogſheads, &c. did he ſend in all?

DRY MEASURE.

TABLES.

2 Pints	make	1 Quart	*Qt.*
2 Quarts	——	1 Pottle	*Pot.*
2 Pottles	——	1 Gallon	*Gal.*
2 Gallons	——	1 Peck	*Pec.*
4 Pecks	——	1 Buſhel	*Bu.*
4 Buſhels	——	1 Coom	*Coom.*
2 Cooms	——	1 Quarter	*Qr.*
4 Quarters	——	1 Chaldron	*Chal.*
5 Quarters	——	1 Wey or Load	*Wey.*
2 Weys	——	1 Laſt	*Laſt.*

Pints.
8 = 1 Gallon.
16 = 2 = 1 Peck.
64 = 8 = 4 = 1 Buſhel.
256 = 32 = 16 = 4 = 1 Coom.
512 = 64 = 32 = 8 = 2 = 1 Quarter.
2560 = 320 = 160 = 40 = 10 = 5 = 1 Wey.
5120 = 640 = 320 = 80 = 20 = 10 = 2 = 1 Laſt.

Note. A chaldron of coals is 36 buſhels.

By

By this Measure Corn, Seeds, Roots, Salt, Sea-coal, Charcoal, Oysters, &c. and all dry Goods, are measured.

The standard Bushel is 18½ Inches wide, and 8 Inches deep. The Coal ditto, 19½ wide, or about a Quarter greater than the Corn Bushel.

A Winchester bushel of malt contains 2150.42 cubic inches.

EXAMPLES.

Last	Wey	Qrs.	Coom	Bus.	Pec.
36	1	4	1	3	3
91	1	2	0	2	2
95	0	4	1	3	1
86	1	3	1	1	3
71	1	0	1	0	2
40	1	2	1	2	0
423	0	4	0	1	3

Qrs.	Bus.	Pec.	Gal.	Qts.	Pts.
12	7	3	1	3	1
11	5	2	1	2	1
98	4	1	0	3	1
25	3	2	1	0	1
8	2	1	1	3	0
0	6	0	1	2	1
157	6	1	0	3	1

Wey	Qr.	Bus.	Pec.	Pot.	Qts.
93	4	7	3	3	1
91	1	4	2	2	0
73	3	2	1	0	1
59	2	3	1	1	0
27	0	0	0	3	1
0	4	6	3	0	1

Qrs.	Bus.	Pec.	Gal.	Pts.
80	6	3	1	3
89	5	2	0	5
46	2	1	1	2
37	7	3	1	1
18	3	2	1	6
0	4	1	0	7

Last	Wey	Qrs.	Coom	Bus.	Pec.
99	1	4	1	3	3
65	1	3	1	2	1
49	0	2	0	1	2
83	1	2	1	3	3
16	0	0	0	2	0

Last	Qrs.	Bus.	Pec.	Gal.
72	6	7	2	1
37	9	6	3	1
68	4	2	1	0
38	3	0	2	0
17	7	5	3	1

A Corn Merchant exported 18 Lasts, 2 Qrs. 5 Buf. of Wheat; 29 Lasts, 6 Qrs. 7 Buf. of Rye; 15 Lasts, 9 Qrs. 3 Buf. of Beans; and 46 Lasts, 6 Buf. of Oats. How many Lasts, &c. were exported in the whole?

TIME.

TIME.

TABLES.

Seconds				*Sc.* or''
60 Seconds	make	1	Minute	*M.* or'
60 Minutes	——	1	Hour	*Hr.*
24 Hours	——	1	Day	*Day.*
7 Days	——	1	Week	*Wk.*
4 Weeks	——	1	Month	*Mo.*
13 Months, 1 Day, 6 Hours, or 365 Days, 6 Hours	}	1	Julian Year	*Yr.*

Seconds

60 =	1 Minute.				
3600 =	60 =	1 Hour.			
86400 =	1440 =	24 =	1 Day.		
604800 =	10080 =	168 =	7 =	1 Week.	
2419200 =	40320 =	672 =	28 =	4 =	1 Month.
31557600 =	525960 =	8766 =	$365\frac{1}{4}$ =	1 Year.	

	Wk.	*Day*	*Hr.*		*Mo.*	*Day*	*Hr.*		*Year.*
or,	52	1	6	=	13	1	6	=	1

N. B. The 1 Day and 6 Hours are commonly neglected; and 13 Months reckoned as a Year.

EXAMPLES.

Yr.	*Mo.*	*Wk.*	*Day*	*H.*	*Mi.*	*Sec.*	*Mo.*	*Wk.*	*Day*
76	8	3	6	20	37	40	19	2	6
57	11	2	3	17	20	35	6	1	4
34	9	3	5	21	16	34	22	3	5
57	6	1	2	16	27	46	7	2	3
35	10	2	4	22	19	52	2	1	6
56	9	3	3	19	22	16	17	3	2
20	6	1	2	21	31	37	11	3	4
339	11	2	4	138	56	20	88	3	2

Days

Days	*H.*	*M.*	*Sec.*
224	14	48	20
365	5	48	55
87	23	15	39
686	23	30	0
79	7	48	0
15	22	41	14
4	12	25	12

Yr.	*M.*	*W.*	*Day*	*H.*	*Mi.*
39	10	2	4	23	58
56	3	1	5	20	50
39	11	3	6	18	10
86	8	2	3	7	0
12	7	1	2	13	33
8	1	0	5	13	26
7	5	2	3	14	21

Mo.	*Wk.*	*D.*	*Hrs.*
47	2	3	20
89	3	1	19
12	2	5	18
27	3	6	12
19	1	4	3
8	0	0	1

Wk.	*Days*	*Hrs.*	*Mi.*	*Sec.*
10	6	16	32	9
8	5	3	42	36
7	3	13	13	42
6	1	18	27	24
3	2	17	41	22
2	1	21	18	27

COMPOUND SUBTRACTION.

COMPOUND SUBTRACTION teaches to find the difference between any two numbers of different denominations.

RULE*.

1. Place the less number under the greater, so that those parts which are of the same denomination may stand directly under each other; and draw a line below them.

2. Begin at the right-hand, and take each number in the lower line from that above it, and set the remainder under it.

3. If any number in the lower line be greater than that above it, increase the upper number by as many as make one of the next higher denomination; then subtract the

* The reason of this rule will readily appear from what has been said in simple subtraction; for the borrowing depends upon the same principle, and is only different, as the numbers to be subtracted are of different denominations.

lower number from the upper one, and ſet down the remainder.

4. Carry the unit borrowed to the next number in the lower line, which ſubtract from the number above it, as before; and proceed in like manner till the whole is finiſhed; and the ſeveral remainders taken together, will be the whole difference required.

The method of proof is the ſame as in ſimple ſubtraction.

EXAMPLES OF MONEY.

	£.	s.	d.		£.	s.	d.		£.	s.	d.
From	9	8	6½	From	16	12	8¾	From	21	13	4¾
Take	4	3	4¼	Take	10	11	6¼	Take	18	9	8½
Rem.	5	5	2¼		6	1	2½		3	3	8¼

	£.	s.	d.		£.	s.	d.		£.	s.	d.
From	136	12	3	From	386	2	7	From	860	0	7¼
Take	95	15	2	Take	197	8	7	Take	99	12	8½
Rem.	40	17	1		188	14	0		760	7	10¾
Proof	136	12	3	Proof	386	2	7	Proof	860	0	7¼

From	45	16	9¾	From	8	12	10½	From	453	6	2¼
Take	13	8	5½	Take	5	16	9¼	Take	165	12	10¾

From 15l. 7s. 10d. take 6l. 4s. 5d. *Anſ.* 9l. 3s. 5d.
From 284l. 9s. 8d. take 192l. 19s. 3d. *Anſ.* 91l. 10s. 5d.
From 2474l. 6¼d. take 1972l. 17s. 7½d.
Anſ. 501l. 2s. 10¾d.

A tradeſman had owing to him 849l. 6s. 8¾d. and received at one time 56l. 2s. 6d. at another 32l. 17s. 5½d. at a third, 101l. 6s. 2d. What remains due to him?

TROY WEIGHT.

	lb.	oz.	dwt.	gr.		lb.	oz.	dwt.	gr.
From	7	3	14	11	From	18	9	10	8
Take	4	2	10	9	Take	9	10	15	20
Rem.	3	1	4	2		8	10	14	12

	lb.	*oz.*	*dwt.*	*gr.*
From	273	0	0	0
Take	98	10	18	21
Rem.	174	1	1	3

	lb.	*oz.*	*dwt.*	*gr.*
From	8	7	17	21
Take	6	2	13	9

	lb.	*oz.*	*dwt.*	*gr.*
From	25	6	0	8
Take	16	8	12	15

	lb.	*oz.*	*dwt.*	*gr.*
From	436	0	16	0
Take	119	6	9	18

From 637 *lb.* 9 *oz.* 8 *gr.* take 288 *lb.* 1 *oz.* 9 *dwt.* 20 *gr.*

Anſ. 348 *lb.* 10 *oz.* 10 *dwt.* 12 *gr.*

From 8947 *lb.* take 5398 *lb.* 6 *oz.* 18 *dwt.* 12 *gr.*

Anſ. 3548 *lb.* 5 *oz.* 1 *dwt.* 12 *gr.*

APOTHECARIES WEIGHT.

	℔	℥	ʒ	℈	*gr.*
From	24	8	7	2	18
Take	17	7	6	1	13
	7	1	1	1	5

	℔	℥	ʒ	℈	*gr.*
From	8	4	7	0	14
Take	0	8	7	2	19
	7	7	7	0	15

	℔	℥	ʒ	℈	*gr.*
From	20	5	6	2	10
Take	13	9	7	1	14
	6	7	7	0	16

	℔	℥	ʒ	℈	*gr.*
From	33	9	6	2	18
Take	17	6	5	1	12

	℔	℥	ʒ	℈	*gr.*
From	8	3	2	1	7
Take	0	10	7	0	15

	℔	℥	ʒ	℈	*gr.*
From	46	0	0	0	0
Take	17	8	3	2	13

AVOIRDUPOIS WEIGHT.

	T.	*cwt.*	*qr.*	*lb.*	*oz.*
From	7	14	1	3	6
Take	2	6	3	4	11
	5	7	1	16	11

	cwt.	*qr.*	*lb.*	*oz.*	*dr.*
From	14	2	12	10	8
Take	6	3	16	15	3
	7	2	23	11	5

	T.	*cwt.*	*qr.*	*lb.*	*oz.*
From	121	14	2	20	14
Take	96	12	1	24	9

	lb.	*oz.*	*dr.*
From	74	3	8
Take	15	6	10
	58	12	14

	cwt.	*qrs.*	*lb.*	*oz.*	*dr.*
From	17	1	0	9	12
Take	8	2	23	12	13

	lb.	*oz.*	*dr.*
From	56	2	0
Take	13	9	11

Bought 2 Tun 5 Cwt. 1 qr. 7 lb. of Sugar; and ſold 1 Tun 19 Cwt. 20 lb. What remains? *Anſ.* 6 *cwt.* 15 *lb.*

WOOL WEIGHT.

	La.	*Sa.*	*W.*	*T.*	*St.*	*Cl.*
From	18	7	1	5	1	1
Take	9	10	1	6	1	1
	8	8	1	4½	0	0

	S.	*W.*	*T.*	*St.*
From	37	1	3	0
Take	18	1	5	1
	18	1	3½	1

	La.	*Sa.*	*W.*	*T.*	*St.*	*Cl.*
From	28	9	1	4	1	0
Take	16	11	1	6	1	1

	W.	*T.*	*St.*	*Cl.*	*lb.*
From	98	4	1	0	5
Take	63	5	1	1	6
	34	4½	1	0	6

	W.	*T.*	*St.*	*Cl.*	*lb.*
From	100	0	0	0	0
Take	40	5	1	1	5

	Sa.	*W.*	*T.*	*St.*
From	45	1	6	1
Take	68	1	3	0

LONG MEASURE.

	Lea.	*Mil.*	*Fur.*	*P.*	*Yd.*
From	20	2	7	38	4
Take	10	1	6	9	3
	10	1	1	29	1

	Yds.	*Ft.*	*In.*	*Bar.*
From	200	2	10	1
Take	59	2	11	2
	140	2	10	2

	Mil.	*Fur.*	*Pl.*	*Yd.*
From	100	2	6	4
Take	19	6	30	3
	80	3	16	1

	Mil.	*Fur.*	*Pl.*	*Yd.*
From	387	0	19	2
Take	209	6	32	4
	177	1	26	$2\frac{1}{2}$

	Lea.	*M.*	*F.*	*P.*	*Yd.*
From	160	1	3	20	2
Take	84	2	6	28	3

	M.	*F.*	*P.*	*Yd.*	*Ft.*	*In.*
From	70	7	13	1	1	2
Take	20	0	14	2	2	8

From 50 Lea. 2 M. 1 Fur. take 19 Lea. 18 Pls. 4 Yds.
Anſ. 31 *Lea.* 2 *Mi.* 21 *Pls.* $1\frac{1}{2}$ *Yd.*

From 79 M. 4 Fur. take 12 M. 6 Fur. 3 Yds. 2 Ft.

CLOTH MEASURE.

	F. E.	*qrs.*	*N.*	*In.*
From	65	1	3	1
Take	13	2	1	2
	51	2	1	$1\frac{1}{4}$

	Yds.	*qrs.*	*N.*	*In.*
From	85	2	1	2
Take	17	3	2	1
	67	2	3	1

	En.E.	*qrs.*	*Nls.*	*In.*
From	204	2	3	1
Take	86	4	2	2
	117	3	0	$1\frac{1}{4}$

	Fr.E.	*qrs.*	*Nl.*	*In.*
From	536	2	1	2
Take	182	5	3	1
	353	2	2	1

	Fl.E.	*qr.*	*Nl.*	*In.*
From	260	1	0	1
Take	150	2	2	2

	Yds.	*qrs.*	*Nl.*	*In.*
From	365	2	1	1
Take	78	3	2	2

From 156 Eng. Ells, take 30 Eng. Ells 1 qr. 1 Nl.
Anſ. 125 *Ells* 3 *qrs.* 3 *Nls.*

From 908 Fr. Ells, take 170 Fr. Ells 4 qrs. 3 Nls.
Anſ. 737 *Fr. Ells* 1 *qr.* 1 *Nl.*

From 856 Yds. take 200 Yds. 2 qrs. 1 Nl. 1 In.
Anſ. 655 *Yds.* 1 *qr.* 2 *Nl.* $1\frac{1}{4}$ *In.*

SQUARE

SQUARE MEASURE.

	Ac.	*Rd.*	*Pl.*	*Yds.*	*Ft.*
From	29	3	26	16	5
Take	12	2	18	20	6
	17	1	7	$25\frac{1}{4}$	8

	Ac.	*Rd.*	*Pl.*	*Yd.*
From	47	2	10	10
Take	23	3	20	4
	23	2	30	6

	Ac.	*Rd.*	*Pl.*	*Yd.*	*Ft.*
From	69	2	13	14	7
Take	30	3	28	30	4

	Ac.	*Rd.*	*Pl.*	*Yd.*
From	200	3	19	13
Take	163	3	38	10
	36	3	21	3

	Ac.	*Rd.*	*Pl.*	*Yd.*	*Ft.*
From	783	0	30	23	1
Take	186	3	36	27	2

	Ac.	*Rd.*	*Pl.*	*Yd.*
From	576	0	0	0
Take	238	2	30	3

From 780 Ac. 2 Rds. take 396 Ac. 3 Rds. 15 Pls.
Anſ. 383 *Ac.* 2 *Rds.* 25 *Pls.*

From 800 Ac. take 100 Ac. 2 Rds. 8 Ft.
Anſ. 799 *Ac.* 1 *Rd.* 39 *Pl.* $29\frac{1}{4}$ *Yd.* 1 *Ft.*

WINE MEASURE.

	Tun.	*Pipe*	*Hbd.*	*Gal.*	*Qt.*
From	436	1	0	20	2
Take	248	1	1	50	3
	187	1	0	32	3

	Hbd.	*Gal.*	*Qt.*	*Pt.*
From	600	40	2	1
Take	193	60	3	1
	406	42	3	0

	Tun.	*Pun.*	*Tier.*	*Gal.*	*Qt.*
From	61	1	0	36	3
Take	18	2	1	31	2
	42	1	0	37	1

	Tun	*Pipe*	*Hbd.*	*Gal.*	*Qt.*
From	200	1	0	30	1
Take	156	1	1	48	2

	Hbd.	*Gal.*	*Qt.*	*Pt.*
From	367	20	2	0
Take	148	8	3	1

	Tun	*Pun.*	*Tier.*	*Gal.*
From	209	1	1	25
Take	131	2	1	38

From 6 Tuns, take 3 Hhds. 15 Gal. 3 Qts.

Ans. 5 *Tun* 47 *Gal.* 1 *Qt.*

From 28 Tun 1 Pun. take 15 Tun 1 Tier. 19 Gal.

Ans. 13 *Tun* 23 *Gal.*

ALE AND BEER MEASURE.

	Butt	Hhd.	Bar.	Kil.	B.F.	Gal.
From	12	1	1	1	1	6
Take	8	1	1	1	1	7
	3	1	½	1	1	7

	Hhd.	B.Fir.	Gal.	Qt.
From	145	2	6	2
Take	98	3	7	3
	46	4	7	3

	Hhd.	Gal.	Qts.	Pt.
From	200	0	0	0
Take	87	50	2	1
	112	3	1	1

	Tuns	Butts	Hhd.	Gal.	Qts.
From	78	1	1	13	0
Take	60	1	1	48	3

	Hhd.	Kil.	Gal.	Qts.	Pt.
From	100	1	12	1	1
Take	40	2	16	3	0

	Pun.	Fir.	Gal.	Qt.	Pt.
From	84	5	3	2	0
Take	26	7	6	1	1

From 12 Tuns 1 Butt, take 8 Tuns 50 Gal. 3 Qts.

Ans. 4 *Tuns* 1 *Hhd.* 3 *Gal.* 1 *Qt.*

From 19 Pun. 1 Hhd. take 10 Pun. 1 Hhd. 40 Gal.

Ans. 1 *Pun.* 1 *Kil.* 14 *Gal.*

DRY MEASURE.

	Last	Wey	Qrs.	Com	Bus.
From	136	1	2	1	2
Take	97	1	3	1	3
	38	1	3	1	3

	Qrs.	Bus.	Pec.	Gal.
From	28	5	1	1
Take	19	6	3	0
	8	6	2	1

	La.	Qrs.	Bus.	Pec.	Gal.
From	91	5	3	2	0
Take	67	8	4	3	1
	23	6	6	2	1

	Wey	Qrs.	Bus.	Pec.
From	86	2	3	2
Take	42	4	6	3
	43	2	4	3

From

	Wey	*Qrs.*	*Buſ.*	*Pec.*
From	12	1	3	2
Take	8	4	2	3

	Wey	*Qrs.*	*Buſ.*	*Pec.*
From	100	3	2	3
Take	86	4	5	2

	Laſt	*Wey*	*Qrs.*	*Coom*
From	65	0	3	1
Take	46	1	3	1

	Qrs.	*Buſ.*	*Pec.*	*Pot.*
From	79	1	1	0
Take	34	2	1	3

From 20 Weys or Loads, take 8 Loads 3 Qrs. 2 Pec.
Anſ. 11 *Loads* 1 *Qr.* 7 *Buſ.* 2 *Pec.*

From 8 Loads 2 Qrs. 1 Coom, take 4 Qrs. 3 Buſ. 2 Pec.
Anſ. 7 *Loads* 3 *Qrs.* 2 *Pec.*

TIME.

	Mo.	*Wk.*	*Dy.*	*Hr.*	*Min.*
From	38	2	3	7	10
Take	10	3	2	10	30
	27	3	0	20	40

	Yr.	*Mo.*	*Wk.*	*D.*
From	176	8	3	4
Take	98	9	2	6
	77	12	0	5

	Mo.	*Wk.*	*Dy.*	*Hr.*	*Mi.*
From	12	1	2	14	12
Take	7	2	3	9	50

	Mo.	*Wk.*	*Dy.*	*Hr.*
From	93	2	1	0
Take	45	3	4	12
	47	2	3	12

	Yrs.	*Mo.*	*Wk.*	*Dy.*
From	1650	9	2	3
Take	486	2	3	5

	Mo.	*Wk.*	*Dy.*	*Hr.*
From	18	0	4	10
Take	9	2	5	21

From 400 Years, take 98 Years 3 Mo. 8 Hr. 10 Sec.
Anſ. 301 *Yrs.* 9 *Mo.* 3 *Wk.* 6 *Dys.* 15 *Hr.* 59′, 50″.

From 87 Months, take 43 Mo. 2 Wks. 3 Dys. 1 Hr.
Anſ. 43 *Mo.* 1 *Wk.* 3 *Dys.* 23 *Hrs.*

From 39 Weeks, take 13 Wks. 6 Dys. 20 Hrs. 11 Min. 13 Sec.
Anſ. 25 *Wks.* 3 *Hrs.* 48 *Min.* 47 *Sec.*

COM-

COMPOUND MULTIPLICATION.

COMPOUND MULTIPLICATION teaches to find the amount of any given number of different denominations repeated a certain propoſed number of times.

RULE*.

1. Place the multiplier under the loweſt denomination of the multiplicand.

2. Multiply the number in the loweſt denomination by the multiplier, and find how many integers of the next higher denomination are contained in the product, and write down what remains.

3. Carry the integers, thus found, to the product of the next higher denomination, with which proceed as before; and ſo on, through all the denominations to the higheſt; and this product, together with the ſeveral remainders, taken as one number, will be the whole amount required.

The method of proof is the ſame as in ſimple multiplication.

EXAMPLES OF MONEY.

1. 9 *lb.* of tobacco at 2 *s.* 8½ *d. per lb.*

	2 *s.*	8½ *d.*
		9
1 *l.*	4 *s.*	4½ *Anſwer.*

2. 3 *lb.* of green tea at 9 *s.* 6 *d. per lb.* *Anſ.* 1 *l.* 8 *s.* 6 *d.*
3. 5 *lb* of loaf ſugar at 1 *s.* 3 *d per lb.* *Anſ.* 6 *s.* 3 *d.*
4. 9 *cwt.* of cheeſe at 1 *l.* 11 *s.* 5 *d. per cwt.* *Anſ.* 14 *l.* 2 *s.* 9 *d.*
5. 12 gallons of brandy at 9 *s.* 6 *d. per gall.* *Anſ.* 5 *l.* 14 *s.*

* The product of a number conſiſting of ſeveral parts, or denominations, by any ſimple number whatever, will, evidently, be expreſſed by taking the product of that ſimple number, and each part by itſelf, as ſo many diſtinct queſtions: thus 25 *l.* 12 *s.* 6 *d.* multiplied by 9, will be 225 *l.* 108 *s.* 54 *d.* = (by taking the ſhillings from the pence, and the pounds from the ſhillings, and placing them in the ſhillings and pounds reſpectively) 230 *l.* 12 *s.* 6 *d.* which is the ſame as the rule: and this will be true when the multiplicand is any compound number whatever.

CON-

CONTRACTIONS.

I. If the multiplier exceed 12, multiply successively by its component parts, instead of the whole number at once.

EXAMPLES.

1. 16*cwt.* of cheese at 1*l.* 18*s.* 8*d.* *per cwt.*

1*l.*	18*s.*	8*d.*
		4
7 -	14 -	8
		4
30*l.*	18*s.*	8*d.* *the answer.*

2. 28 yards of broad cloth at 19*s* 4*d.* *per yd.* *Ans.* 27*l.* 1*s.* 4*d.*
3. 35 firkins of butter at 15*s.* 3½*d.* *per firkin.* *Ans.* 26*l.* 15*s.* 2½*d.*
4. 42*cwt.* of tallow at 34*s.* 6*d.* *per cwt.* *Ans.* 72*l.* 9*s.*
5. 64 gallons of brandy at 9*s.* 6*d.* *per gallon.* *Ans.* 30*l.* 8*s.*
6. 96 quarters of rye at 1*l.* 3*s.* 4*d.* *per quarter.* *Ans.* 112*l.*
7. 120 dozen of candles at 5*s.* 9*d.* *per doz.* *Ans.* 34*l.* 10*s.*
8. 132 yards of Irish cloth at 2*s.* 4*d.* *per yd.* *Ans.* 15*l.* 8*s.*
9. 144 reams of paper at 13*s.* 4*d.* *per ream.* *Ans.* 96*l.*
10. 1210 yards of shalloon at 2*s.* 2*d* *per yard.* *Ans.* 131*l.* 1*s.* 8*d.*

II. If the multiplier cannot be produced by the multiplication of simple numbers, take the nearest number to it, either greater or less, which can be so produced, and multiply by its parts as before.

Then multiply the multiplicand by the difference between this number and the multiplier, and add or subtract the product from that before found, according as the given number was greater or less than the assumed one.

EXAM-

EXAMPLES.

1. 17 ells of holland at 7*s.* $8\frac{1}{2}$*d.* *per ell.*

```
        7s.  8½d.
              4
    -------------
    1 - 10 - 10
              4
    -------------
    6 -  3 - 4
         7 - 8½
    -------------
```

6*l.* 11*s.* $0\frac{1}{2}$*d.* *the answer.*

2. 23 ells of dowlas at 1*s.* $6\frac{1}{2}$*d.* *per ell.*
Ans. 1*l.* 15*s.* $5\frac{1}{2}$*d.*

3. 46 bushels of wheat at 4*s.* $7\frac{1}{4}$*d.* *per bushel.*
Ans. 10*l.* 11*s.* $9\frac{1}{2}$*d.*

4. 59 yards of tabby at 7*s.* 10*d.* *per yard.*
Ans. 23*l.* 2*s.* 2*d.*

5. 94 pair of silk stockings at 12*s.* 2*d.* *per pair.*
Ans. 57*l.* 3*s.* 8*d.*

6. 117 *cwt.* of malaga raisins at 1*l.* 2*s.* 3*d.* *per cwt.*
Ans. 130*l.* 3*s.* 3*d.*

EXAMPLES OF WEIGHTS, MEASURES, &c.

lb.	*oz.*	*dwt.*	*gr.*
21	1	7	13
			4

lb.	*oz.*	*dr.*	*sc.*	*gr.*
2	4	2	1	-
				7

cwt.	*qr.*	*lb.*	*oz.*
27	1	13	12
			12

mls.	*fu.*	*pls.*	*yds.*
24	3	20	2
			6

yds.	*qrs.*	*na.*
127	2	2
		8

ac.	*ro.*	*po.*
27	2	1
		9

tuns.	*hhd.*	*gal.*	*pts.*
29	1	20	3
			5

we.	*qr.*	*bu.*	*pe.*
27	1	7	2
			7

mo.	*we.*	*da.*	*ho.*	*min.*
175	3	6	20	59
				11

COMPOUND DIVISION.

COMPOUND DIVISION teaches to find how often one given number is contained in another of different denominations.

RULE*.

1. Place the divisor and dividend as in simple division.
2. Begin at the left-hand, or highest denomination of the dividend, which divide by the divisor, and write down the quotient.
3. If there be any remainder after this division, find how many integers of the next lower denomination it is equal to, and add them to the number, if any, which stands in that denomination.
4. Divide this number, so found, by the divisor, and write the quotient under its proper denomination.
5. Proceed in the same manner through all the denominations to the lowest, and the whole quotient, thus found, will be the answer required.

The method of proof is the same as in simple division.

EXAMPLES OF MONEY.

1. Divide 225 *l.* 2 *s.* 4 *d.* by 2.

2)225 *l.* 2 *s.* 4 *d.*

112 *l.* 11 *s.* 2 *d.* *the quotient.*

2. Divide	751 *l.*	14 *s.*	$7\frac{3}{4}$ *d.*	by 3.	*Ans.*	250 *l.*	11 *s.*	$6\frac{1}{2}$ *d.*
3. Divide	821 *l.*	17 *s.*	$9\frac{3}{4}$ *d.*	by 4.	*Ans.*	205 *l.*	9 *s.*	$5\frac{1}{4}$ *d.*
4. Divide	2382 *l.*	13 *s.*	$5\frac{1}{2}$ *d.*	by 5.	*Ans.*	476 *l.*	10 *s.*	$8\frac{1}{4}$ *d.*
5. Divide	28 *l.*	2 *s.*	$1\frac{1}{2}$ *d.*	by 6.	*Ans.*	4 *l.*	13 *s.*	$8\frac{1}{4}$ *d.*
6. Divide	55 *l.*	14 *s.*	$\frac{3}{4}$ *d.*	by 7.	*Ans.*	7 *l.*	19 *s.*	$1\frac{3}{4}$ *d.*
7. Divide	6 *l.*	5 *s.*	4 *d.*	by 8.	*Ans.*		15 *s.*	8 *d.*
8. Divide	135 *l.*	10 *s.*	7 *d.*	by 9.	*Ans.*	15 *l.*	1 *s.*	2 *d.*
9. Divide	21 *l.*	18 *s.*	4 *d.*	by 10.	*Ans.*	2 *l.*	3 *s.*	10 *d.*
10. Divide	227 *l.*	10 *s.*	5 *d.*	by 11.	*Ans.*	20 *l.*	13 *s.*	8 *d.*
11. Divide	1332 *l.*	11 *s.*	$8\frac{1}{2}$ *d.*	by 12.	*Ans.*	111 *l.*	0 *s.*	$11\frac{1}{2}$ *d.*

CON-

* To divide a number consisting of several denominations by any simple number whatever, is, evidently, the same as dividing all the parts or members of which that number is composed by the same simple number. And this will be true when any of the parts are not an exact multiple

CONTRACTIONS.

I. If the divisor exceed 12, find what simple numbers, multiplied together, will produce it, and divide by them separately, as in simple division.

EXAMPLES.

1. What is cheese *per cwt.* if 16 *cwt.* cost 30*l.* 18*s.* 8*d.*?

4)30*l.* 18*s.* 8*d.*

4)7*l.* 14*s.* 8*d.*

1*l.* 18*s.* 8*d.* *the answer.*

2. If 20 *cwt.* of tobacco comes to 120*l.* 10*s.* what is that *per cwt.*? *Ans.* 6*l.* 0*s.* 6*d.*
3. Divide 57*l.* 3*s.* 7*d.* by 35. *Ans.* 1*l.* 12*s.* 8*d.*
4. Divide 85*l.* 6*s.* by 72. *Ans.* 1*l.* 3*s.* $8\frac{1}{4}$*d.*
5. Divide 31*l.* 2*s.* $10\frac{1}{2}$*d.* by 99. *Ans.* 6*s.* $3\frac{1}{2}$*d.*
6. At 18*l.* 18*s.* *per cwt.* how much *per lb.*? *Ans.* 3*s.* $4\frac{1}{2}$*d.*

II. If the divisor cannot be produced by the multiplication of small numbers, divide by the whole divisor at once, after the manner of long division.

EXAMPLES.

1. Divide 74*l.* 13*s.* 6*d.* by 17

```
17) 74 l. 13 s. 6 d. (4 l. 7 s. 10 d.
    68
    --
     6
    20
    --
   133
   119
   ---
    14
    12
   ---
   174
   170
   ---
     4
```

2. Divide

tiple of the divisor: for by conceiving the number, by which it exceeds that multiple, to have its proper value, by being placed in the next lower denomination, the dividend will still be divided into parts, and the true quotient

2. Divide 23*l.* 15*s.* $7\frac{1}{4}$*d.* by 37. *Anſ.* 12*s.* $10\frac{1}{4}$*d.*
3. Divide 199*l.* 3*s.* 10*d.* by 53. *Anſ.* 3*l.* 15*s.* 2*d.*
4. Divide 675*l.* 12*s.* 6*d.* by 138. *Anſ.* 4*l.* 17*s.* 11*d.*
5. Divide 315*l.* 3*s.* $10\frac{1}{4}$*d.* by 365. *Anſ.* 17*s.* $3\frac{1}{4}$*d.*

EXAMPLES OF WEIGHTS AND MEASURES.

1. Divide 23*lb.* 7*oz.* 6*dwts.* 12*gr.* by 7.
Anſ. 3*lb.* 4*oz.* 9*dwts.* 12*gr.*
2. Divide 13*lb.* 1*oz.* 2*dr.* -*ſcr.* 10*gr.* by 12.
Anſ. 1*lb.* 1*oz.* 0*dr.* 2*ſcr.* 10*gr.*
3. Divide 1061*cwt.* 2*qr.* by 28.
Anſ. 37*cwt.* 3*qrs.* 18*lb.*
4. Divide 375*mi.* 2*fur.* 7*po.* 2*yds.* 1*f.* 2*in.* by 39.
Anſ. 9*mi.* 4*fur.* 39*po.* -*yds.* 2*fe.* 8*in.*
5. Divide 571*yds.* 2*qrs.* 1*na.* by 47.
Anſ. 12*yds.* -*qrs.* 2*na.*
6. Divide 51*ac.* 2*ro.* 3*po.* by 51.
Anſ. 1*ac.* -*ro.* 1*po.*
7. Divide 10*tu.* 2*hhds.* 17*gall.* 2*pi.* by 67.
Anſ. 39*galls.* 3*pi.*
8. Divide 120*la.* 1*qrs.* 1*bu.* 2*pe.* by 74.
Anſ. 1*la.* 6*qrs.* 1*bu.* 3*pe.*
9. Divide 120*mo.* 2*we.* 3*da.* 5*ho.* 20*min.* by 111.
Anſ. 1*mo.* 0*we.* 2*da.* 10*ho.* 12*mi.*

REDUCTION.

REDUCTION teaches to bring numbers from one name or denomination to another, without changing their value.

RULE *.

I. When the numbers are to be reduced from a higher denomination to a lower.

1. Multiply the number in the higher denomination by as many of the next lower as make an integer, or one, in that higher, and ſet down the product.

2. To

quotient found as before: thus, 25*l.* 12*s.* 3*d.* divided by 9, will be the ſame as 18*l.* 144*s.* 99*d.* divided by 9, which is equal to 2*l.* 16*s.* 11*d.* as by the rule; and the method of carrying from one denomination to another is exactly the ſame.

* The reaſon of this rule is exceedingly obvious; for pounds are brought into ſhillings by multiplying them by 20; ſhillings into pence by

2. To this product add the number, if any, which was in this lower denomination before; and multiply the ſum by as many of the next lower denomination as make an integer in the preſent one.

3. Proceed in the ſame manner through all the denominations to the loweſt, and the number laſt found will be the value of all the numbers which were in the higher denominations, taken together.

II. When the numbers are to be reduced from a lower denomination to a higher.

1. Divide the given number by as many of that denomination as make one of the next higher, and ſet down what remains.

2. Divide the quotient by as many of this as make one of the next higher denomination, and ſet down what remains in like manner as before.

3. Proceed in the ſame manner through all the denominations to the higheſt; and the quotient laſt found, together with the ſeveral remainders, if any, will be of the ſame value as the firſt number propoſed.

The method of proof is to work the queſtion back again.

EXAMPLES.

1. In 1465*l.* 14*s.* 5*d.* how many farthings?

$$\begin{array}{r} 1465\,l.\ 14\,s.\ 5\,d. \\ 20 \\ \hline 29314 \\ 12 \\ \hline 351773 \\ 4 \\ \hline 1407092 \end{array}$$ *Anſwer.*

by multiplying them by 12; and pence into farthings by multiplying them by 4; and the contrary by diviſion: and this will be true in the reduction of numbers conſiſting of any denominations whatſoever. In moſt books of practical Arithmetic, this rule is uſually divided into two parts, called Reduction aſcending, and Reduction deſcending; but theſe diſtinctions appear to be totally unneceſſary.

Reduce

Reduce 1407092 farthings into pounds.

4)1407092

12)351773

2,0)2931,4—5

1465*l.* 14*s.* 5*d.* *Answer.*

2. In 12*l.* how many farthings? *Ans.* 11520
3. In 6169 pence how many pounds? *Ans.* 25*l.* 14*s.* 1*d.*
4. In 35 guineas how many farthings? *Ans.* 35280.
5. In 420 quarter-guineas how many moidores? *Ans.* 81 *and* 18*s.*
6. In 231*l.* 16*s.* how many ducats at 4*s.* 9*d.* each? *Ans.* 976.
7. In 274 marks each 13*s.* 4*d.* and 87 nobles each 6*s.* 8*d.* how many pounds? *Ans.* 211*l.* 13*s.* 4*d.*
8. In 1776 quarter-guineas how many six-pences? *Ans.* 18648.
9. Reduce 1776 six-and-thirties to half crowns? *Ans.* $25574\frac{2}{5}$.
10. In 50807 moidores, how many pieces of coin, each 4*s.* 6*d.* *Ans.* 304842.
11. In 213210 grains how many pounds? *Ans.* 37 *lb.* 3 *dwts.* 18 *gr.*
12. In 59 *lb.* 13 *dwts.* 5 *gr.* how many grains? *Ans.* 340157.
13. In 8012131 grains how many pounds? *Ans.* 1390 *lb.* 11 *oz.* 18 *dwts.* 19 *gr.*
14. In 35 *ton.* 17 *cwt.* 1 *qr.* 23 *lb.* 7 *oz.* 13 *dr.* how many drams *Ans.* 20571005.
15. In 37 *cwt.* 2 *qr.* 17 *lb.* how many *lbs.* troy, a *lb.* avoirdupois being equal to 14 *oz.* 11 *dwts.* $15\frac{1}{2}$ *gr.* troy? *Ans.* 5124 *lb.* 5 *oz.* 10 *dwt.* $11\frac{1}{2}$ *gr.*
16. How many barley-corns will reach round the world, supposing it, according to the best calculations, to be 8340 leagues? *Ans.* 4755801600.
17. In 17 pieces of cloth, each 27 flemish ells, how many yards? *Ans.* 344 *yds.* 1 *qr.*
18. How many minutes are there since the birth of Christ to the year 1776, allowing the year to consist of 365 *da.* 5 *ho.* 48 *min.* 58 *sec.*? *Ans.* 934085364 *m.* 48 *sec.*

19. How many seconds are there in a solar year, which consists of 365 days, 5 hours, 48 minutes, and 58 seconds?
20. How long would it require to count ten hundred million of money, at the rate of 100*l.* a minute without intermission?

THE RULE OF THREE DIRECT.

The RULE OF THREE DIRECT teaches from three given numbers to find a fourth; between which and one of those three, there shall be the same proportion as between the other two.

Of the three given numbers, two are called the Terms of Supposition and the other the Term of Demand.

RULE*.

1. State the question; that is place the three given numbers in a straight line, making that Term of Supposition, which is of the same kind with the Term of Demand, the first number, the other Term of Supposition the second, and the Term of Demand the third.

2. If the first and third numbers consist of different denominations, reduce them both to the same: and if the second be a compound number, reduce it to the lowest denomination mentioned.

3. Multiply the second and third numbers together, and divide the product by the first; and the quotient, if there be no remainder, is the answer, or fourth number required; which will be of the same denomination as the second number was reduced to.

4. If after division there be any remainder, reduce it to the next denomination below that which the second number was reduced to, and divide by the same divisor as before, and the quotient will be of this last denomination.

5. Proceed

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* This rule, on account of its great and extensive usefulness, is frequently called THE GOLDEN RULE OF PROPORTION. It is founded on this obvious principle, that the magnitude or quantity of any effect varies constantly in proportion to the varying part of the cause: thus, the quantity of goods bought is in proportion to the money laid out; the space gone over by an uniform motion is in proportion to the time, &c. The truth of the rule, as applied to ordinary enquiries, may be made

5. Proceed thus with all the remainders, till you have reduced them to the lowest denomination which the second number admits of, and the several quotients taken together, will be the answer required.

N. B. Two or more statings are sometimes necessary, which may always be known from the nature of the question.

The method of proof is by reversing the question, or working it back again.

EXAMPLES.

1. If 2 *cwt.* 3 *qrs.* 23 *lb.* of raisins cost 6 *l.* 1 *s.* 8 *d.* what will 12 *cwt.* 2 *qrs.* cost at the same rate?

```
     cwt. qrs. lb.      £.  s.  d.      cwt. qrs.
If    2    3   23   :   6   1   8   ::   12   2
      4                 20                4
     ---               ---               ---
     11                121               50
     28                 12               28
     ---               ----              ---
     331 lb.           1460 d.          1400 lb.
     ---               1400              ---
                     --------
                      584000
                     1460
                    -------- (12)
            331)2044000(6175 d.
                1986    ----
                ----  20)514 s.—7 d.
                  580    ----
                  331   25 l.—14 s.—7 d. Ans.
                  ---
                  2490
                  2317
                  ----
                   1730
                   1655
                   ----
                    75 rem.
```

2. What

made sufficiently evident, by attending only to principles already explained.—It is shewn in multiplication of money, that the price of one multiplied by the quantity, is the price of the whole; and in division, that the price of the whole, divided by the quantity, is the price of one. Now, in all cases of valuing goods, &c. where one is the first term of the

2. What is the value of a *cwt.* of sugar at $5\frac{1}{2}$*d. per lb.* *Anſ.* 2*l.* 11*s.* 4*d.*

3. What is the value of a chaldron of coals at $11\frac{1}{2}$*d. per buſhel?* *Anſ.* 1*l.* 14*s.* 6*d.*

4. At $10\frac{1}{2}$*d. per lb.* what is the value of a firkin of butter containing 56*lb.?* *Anſ.* 2*l.* 9*s.*

5. What is the value of a pipe of wine at $10\frac{1}{2}$*d. per pint?* *Anſ.* 44*l.* 2*s.*

6. At 3*l.* 9*s. per cwt.* what is the value of a pack of wool weighing 2*cwt.* 2 *qrs.* 13 *lb.?* *Anſ.* 9*l.* 0*s.* 6*d.*$\frac{12}{112}$

7. What is the value of $1\frac{1}{2}$ *cwt.* of coffee at $5\frac{1}{2}$*d. per oz.?* *Anſ.* 61*l.* 12*s.*

8. What is the value of $19\frac{1}{2}$ chaldron of coals at 1*l.* 11*s.* 6*d per* chaldron? *Anſ.* 30*l* 14*s.* 3*d.*

9. Bought 3 caſks of raiſins, each weighing 2 *cwt.* 2 *qrs.* 25 *lb.* what will they come to at 2*l.* 1*s.* 8*d. per cwt.?* *Anſ.* 17*l.* 0*s.* $4\frac{3}{4}$*d.*$\frac{32}{112}$

10. What is the value of 2 *qrs.* 1 *na.* of velvet at 19*s.* $8\frac{1}{2}$*d. per Eng. ell?* *Anſ.* 8*s.* $10\frac{1}{4}$*d.*$\frac{14}{20}$

11. Bought 12 pockets of hops, each weighing 1 *cwt.* 2 *qrs.* 17 *lb.*; what do they come to at 4*l.* 1*s.* 4*d. per cwt.?* *Anſ.* 80*l.* 12*s.* $1\frac{1}{2}$*d.*$\frac{96}{112}$

12. What is the tax upon 745*l.* 14*s.* 8*d.* at 3*s.* 6*d.* in the pound? *Anſ.* 130*l.* 10*s.* $0\frac{3}{4}$*d.*$\frac{48}{240}$

13. If $\frac{3}{4}$ of a yard of velvet coſt 7*s.* 3*d.* how many yards can I buy for 13*l.* 15*s.* 6*d?* *Anſ.* 28 *yds.* 2 *qrs.*

14. If an ingot of gold, weighing 9*lb.* 9*oz.* 12 *dwts.* be worth 411*l.* 12*s.* what is that *per grain?* *Anſ.* $1\frac{3}{4}$*d.*

15. How many quarters of corn can I buy for 40 guineas at 4*s. per buſhel?* *Anſ.* 26 *qrs.* 2 *bu.*

16. If 1 *Eng. ell* 2 *qrs.* coſt 4*s.* 7*d.* what will $39\frac{1}{2}$ yards coſt? *Anſ.* 5*l.* 3*s.* $5\frac{1}{4}$*d.*$\frac{5}{7}$

17. What is the value of a pack of wool weighing 2 *cwt.* 1 *qr.* 19 *lb.* at 8*s.* 6*d. per ſtone?* *Anſ.* 8*l.* 4*s.* $6\frac{1}{4}$*d.*$\frac{10}{14}$

18. Bought

---

the proportion, it is plain that the anſwer found by this rule, will be the ſame as that found by multiplication of money; and where one is the laſt term of the proportion, it will be the ſame as that found by diviſion of money. In like manner, if the firſt term be any number whatever, it is plain that the product of the ſecond and third terms will be greater than the true anſwer required, by as much as the price in the ſecond term exceeds the price of one, or as the firſt term exceeds an unit. Conſequently this product divided by the firſt term, will give the true anſwer required, which is the ſame as the rule.

Direct

18. Bought 4 bales of cloth, each containing 6 pieces, and each piece 27 yards at 16*l.* 4*s.* *per* piece, what is the value of the whole and the rate *per yard?*

Ans. 388*l.* 16*s.* at 12*s.* *per yard.*

19. If an ounce of silver be worth 5*s.* 6*d.* what is the price of a tankard that weighs 1*lb.* 10*oz.* 10*dwts.* 4 *grs.*?

Ans. 6*l.* 3*s.* $9\frac{1}{2}$*d.* $\frac{96}{480}$

20. What does 59*cwt.* 2*qrs.* 24*lb.* of tobacco come to at 2*l.* 14*s.* 5*d.* *per cwt.*? *Ans.* 162*l.* 9*s.* 5*d.* $\frac{48}{112}$

21. What is the half-year's rent of 547 acres of land, at 15*s* 6*d.* *per acre?* *Ans.* 211*l.* 19*s.* 3*d.*

22. At half-a-guinea *per* week, how many months board can I have for 100*l.*? *Ans.* 47 *mo.* 2 *we.* 3 *da.* $\frac{42}{126}$

23. Bought 1000 *Flem. ells* of cloth for 90*l.* how must I sell it *per* ell *English* to gain 10*l.* by the whole?

Ans. 3*s.* 4*d.*

24. Suppose a gentleman's income is 500 guineas a year, and he spends 19*s.* 7*d.* *per* day, one day with another, how much will he have saved at the year's end?

Ans. 167*l.* 12*s.* 1*d.*

25. If $1\frac{3}{4}$ ounce of silver plate cost 10*s.* $11\frac{1}{4}$*d.* what will a service, weighing 327*oz.* 12*dwts.* 9*gr.* cost at that rate?

Ans. 102*l.* 7*s.* $7\frac{1}{4}$*d.* $\frac{525}{840}$

26. At 13*s.* $2\frac{1}{2}$*d.* *per* yard, what is the value of a piece of cloth containing $52\frac{3}{4}$ *eng. ells?*

Ans. 43*l.* 10*s.* 11*d.* 0$\frac{6}{16}$

27. How many *eng. ells* of holland may be bought for 100 guineas at 8*s.* $9\frac{1}{2}$*d.* *per* yard?

Ans. 191 *ells* 0*qr.* 1 *na.* $\frac{338}{422}$

28. What is the value of 172 pigs of lead, each weighing 3*cwt.* 2*qrs.* $17\frac{1}{2}$*lb.* at 8*l.* 17*s.* 6*d.* *per* fother of $19\frac{1}{2}$ *cwt.*?

Ans. 286*l.* 4*s.* $4\frac{1}{2}$*d.*

29. Bought

Direct and inverse proportion are properly only parts of the same general rule, and, in a scientific arrangement, it would be best to consider them in that manner: but I have here preserved the common distinction, because I have observed that young persons in general find them more intelligible.

Note 1. When it can be done, multiply and divide as in compound multiplication and division.

2. If the 1st. term, and either the 2d. or 3d. can be divided by any number without a remainder, let them be so divided, and the quotients used instead of them.

The

29. Bought 25 pieces of holland, each containing 35 *eng. ell's*, for 300 guineas, what is that *per* yard?

Anſ. 8*s.* $0\frac{3}{4}$*d.*$\frac{225}{3125}$

30. If I buy 15 yards of cloth for 11 guineas, how many flemiſh ells can I buy for 240*l.* 13*s.* 4*d.* at the ſame rate?

Anſ. 416 *flem. ells*$\frac{2048}{2772}$

31. The rents of a whole pariſh amount to 1750*l.* and a rate is granted of 32*l.* 16*s.* 6*d.*; what is that in the pound?

Anſ. $4\frac{1}{2}$*d.*$\frac{12}{1750}$

32. If my horſe ſtands me in $11\frac{1}{2}$*d.* *per* day keeping, what will be the charge of 11 horſes for the year?

Anſ. 192*l.* 7*s.* $8\frac{1}{2}$*d.*

33. A perſon breaking, owes in all 1490*l.* 5*s.* 10*d.* and has in money, goods, and recoverable debts 784*l.* 17*s.* 4*d.*: if theſe things be delivered to his creditors, what will they get in the pound? *Anſ.* 10*s.* $6\frac{1}{4}$*d.*$\frac{209930}{357670}$

34. What muſt 40*s.* pay towards a tax, when 652*l.* 13*s* 4*d.* is aſſeſſed at 83*l.* 12*s.* 4*d.*? *Anſ.* 5*s.* $1\frac{1}{4}$*d.*$\frac{15376}{156640}$

35. Bought 3 tons of oil for 151*l.* 14*s.* 85 gallons of which being damaged, I deſire to know how I may ſell the remainder *per* gallon, ſo as neither to gain or loſe by the bargain? *Anſ.* 4*s.* $6\frac{1}{4}$*d.*$\frac{25}{671}$

36. What quantity of water muſt I add to a pipe of mountain wine, value 33*l.* to reduce the firſt coſt to 4*s.* 6*d.* *per gallon*? *Anſ.* 20*gal.* 2*q.* 1*pi.*$\frac{1}{3}$

37. If 15 ells of ſtuff $\frac{3}{4}$ yard wide coſt 37*s.* 6*d.* what will 40 ells of the ſame ſtuff coſt, being yard wide?

Anſ. 6*l.* 13*s.* 4*d.*

38. Shipped for Barbadoes 500 pair of ſtockings at 3*s.* 6*d.* *per* pair, and 1650 yards of baize at 1*s.* 3*d.* *per* yard, and have received in return 348 gallons of rum at 6*s.* 8*d.* *per* gallon, and 750*lb.* of indigo at 1*s.* 4*d.* *per lb.*: what remains due upon my adventure? *Anſ* 24*l.* 12*s.* 6*d.*

The four following methods of operation, when they can be uſed, perform the work in a much ſhorter manner than the general rule.

1. Divide the 2d. term by the 1ſt. and multiply the quotient into the 3d. and the product will be the anſwer.

2. Divide the 3d. term by the 1ſt. and multiply the quotient into the 2d. and the product will be the anſwer.

3. Divide the 1ſt. term by the 2d. and the 3d. by that quotient, and the laſt quotient will be the anſwer.

4. Divide the 1ſt. term by the 3d. and the ſecond by that quotient, and the laſt quotient will be the anſwer.

39. What

39. What is a quarter's rent of 500 acres of land, which is let for 1*l.* 15*s.* 6*d.* an acre *per* annum?

40. A Factor bought 19 pieces of Holland cloth, which cost him 176*l.* 13*s.* at the rate of 5*s.* 3*d.* *per* ell Flemish; how many English ells did the 19 pieces contain?

41. A person failing in trade, compounds with his creditors to pay them half-a-guinea in the pound, and accordingly paid them 1852*l.* 13*s.* 6*d.* what was his whole debt?

42. If an ounce of gold cost 5 guineas, what is the value of one grain?

43. If 3 *cwt.* of tea cost 40*l.* 13*s.* at how much must it be sold *per lb.* to gain 10*l.* by the whole?

44. How many pieces of Holland, each containing 15 ells Flemish, may be bought for 30*l.* 16*s.* 5*d.* at the rate of 5*s.* 3*d.* *per* ell English?

45. If a gentleman's estate be worth 384*l.* 16*s.* a year, and the land-tax be assessed at 2*s.* $9\frac{1}{2}$*d.* *per* pound, what is his net annual income?

46. The circumference of the earth is about 25000 miles; at what rate *per* hour must a body be carried, to pass completely round it in 23 hours 56 minutes, which is the length of a sidereal day?

THE RULE OF THREE INVERSE.

The RULE OF THREE INVERSE, teaches from three numbers given, as before, to find a fourth, between which and one of the Terms of Supposition, there shall be the same proportion as between the Term of Demand and the other term.

RULE*.

Multiply the Terms of Supposition together, and divide by the Term of Demand, and the quotient is the answer or fourth number required.

* The reason of this rule may be explained from the principles of compound multiplication and division, in the same manner as the direct rule. *For example:* If 6 men can do a piece of work in 10 days, in how many days will 12 men do it?

As 6 *men* : 10 *days* :: 12 *men* : $\frac{6 \times 10}{12} = 5$ *days*, the answer. And here the product of the first and second terms, *i. e.* 6 times 10, or 60, is evidently the time in which one man would perform the work; therefore 12 men will do it in one twelfth part of that time, or 5 days; and this reasoning is applicable to any other instance whatever.

Or, having ſtated the queſtion, and reduced the terms as ſhewn in the Rule of Three Direct, multiply the firſt and ſecond numbers together, and divide by the third, and the quotient is the anſwer, in the ſame denomination which the ſecond number was reduced to.

N. B. To diſtinguiſh whether a queſtion belongs to the Rule of Three Direct or Inverſe, obſerve, that, when the queſtion is properly ſtated, if the third term be greater than the firſt, and the nature of the queſtion requires that the fourth term ſhall be greater than the ſecond; or if the third be leſs than the firſt, that the fourth ſhall be leſs than the ſecond, the queſtion belongs to the Rule of Three Direct.

But if the third term be greater than the firſt, and it appears, from conſidering the queſtion, that the fourth muſt be leſs than the ſecond; or if the third be leſs than the firſt, that the fourth muſt be greater than the ſecond, it belongs to the Rule of Three Inverſe.

The method of proof is by reverſing the queſtion.

EXAMPLES.

1. What quantity of ſhalloon that is 3 quarters of a yard wide, will line $7\frac{1}{2}$ yards of cloth, that is $1\frac{1}{2}$ yard wide?

1 *yd.* 2 *qrs.*	:	7 *yds.* 2 *qrs.*	::	3 *qrs.*
4		4		
6		30		
		6		
		3) 180		
		4) 60		
		15 *yards, the anſwer.*		

2. If 100 workmen can finiſh a piece of work in 12 days, how many are ſufficient to do the ſame in 3 days?
Anſ. 400 *men.*

3. How much in length that is $4\frac{1}{2}$ inches broad will make a ſquare foot? *Anſ.* 32 *inches.*

4. How many yards of matting 2 *fe.* 6 *in.* broad will cover a floor that is 27 *fe.* long, and 20 *fe.* broad?
Anſ. 72 *yards.*

5. How many yards of cloth 3 *qrs.* wide, are equal in meaſure to 30 *yds.* 5 *qrs.* wide? *Anſ.* 50 *yards.*

6. A.

6. A. borrowed of his friend B. 250 *l.* for 7 months, promising to do him the like kindneſs: ſome time after B. had occaſion for 300 *l.* how long may he keep it to be made full amends for the favour? *Anſ.* 5 *mo.* 3 *we.* 2 *da.* $\frac{109}{360}$

7. If, when the price of a buſhel of wheat is 6 *s.* 3 *d.* the penny loaf weigh 9 *oz.* what ought it to weigh when wheat is at 8 *s.* $2\frac{1}{2}$ *d. per buſhel?* *Anſ.* 6 *oz.* 13 *dr.* $\frac{122}{197}$

8. How many yards of ſtuff 3 *qrs.* broad will line a cloak that is $5\frac{1}{2}$ *yds.* in length, and $1\frac{1}{4}$ *yd.* broad?
Anſ. 9 *yds.* 2 *qr.* 2 *na.* $\frac{2}{3}$

9. If $4\frac{1}{2}$ *cwt.* may be carried 36 miles for 35 *s.* how many pounds can I have carried 20 miles for the ſame money?
Anſ. 907 *lb.* 3 *oz.* 3 *dr.* $\frac{4}{20}$

10. How much in length that is $13\frac{1}{2}$ poles in breadth muſt be taken to contain an acre?
Anſ. 11 *po.* 4 *yds.* 2 *fe.* 0 *in.* $\frac{18}{27}$

11. How many yards of canvas that is ell wide, will line 20 yards of ſay that is 3 *qrs.* wide? *Anſ.* 12 *yds.*

12. If 30 men can perform a piece of work in 11 days, how many men will accompliſh another piece of work four times as large in a fifth part of the time? *Anſ.* 600

13. A wall that is to be built to the height of 27 feet, was raiſed 9 feet by 12 men in 6 days: how many men muſt be employed to finiſh the wall in 4 days, at the ſame rate of working? *Anſ.* 36 *men.*

14. How many yards of paper $1\frac{1}{4}$ *yd.* wide, will be ſufficient to hang a room, which is 20 *yds.* in circumference, and 4 in height?

15. If 14 *cwt.* be carried 136 miles for 5 *l.* 5 *s.* how many hundred weight may be carried $79\frac{1}{3}$ miles for the ſame money?

16. How many Venetian ducats, at 4 *s.* 4 *d.* each, are equal to 730 rix-dollars, at 4 *s.* $5\frac{3}{4}$ *d.* each?

17. How many yards of canvas, which is 1 Engliſh ell wide, will line 15 French ells of ſay, which is 1 Flemiſh ell wide?

18. If a perſon drink 20 pints of wine *per* month, when it coſt 8 *s.* a gallon, how many pints may he drink in the ſame time, without increaſing the expence, when wine coſts 10 *s. per* gallon?

19. If a taylor can make a coat and waiſtcoat with 3 *yds.* and 3 *qrs.* of cloth which is 1 *yd.* and a half in breadth, how many *yds.* will he require to make the ſame, when the breadth is only 2 *qrs.* 2 *nls.*?

COM-

COMPOUND PROPORTION.

Compound Proportion teaches to resolve such questions as require two or more statings by simple proportion; and that, whether they be direct or inverse.

Rule*.

1. Put all the Terms of Supposition one above the other, in the first place, except that which is of the same nature with the term sought, which put in the second place.

2. Place all the Terms of Demand one above another, in the third place, in the same order as their corresponding Terms of Supposition were put in the first place.

3. The first and third term in every row will then be of the same nature, and must be reduced to one denomination; and the second term, as usual, to the lowest denomination mentioned.

4. Examine every row separately: by considering whether, if the first term required the second, the third would require more or less than the second? If it require *more*, mark the *less* extreme with a cross; but if *less*, mark the *greater* extreme.

5. Multiply all those numbers together which are marked for a divisor, and those which are not marked for a dividend, and the quotient will be the answer sought.

Note, When the same numbers are found in the divisor as in the dividend, they may be thrown out of both.

Examples.

1. If 16 horses can eat up 9 bushels of oats in 6 days, how many horses would eat up 24 bushels in 7 days, at the same rate?

$$\begin{array}{lcccl} + \; 9 \textit{ bushels} & : & 16 \textit{ horses} & :: & 24 \textit{ bushels.} \\ 6 \textit{ days} & : & \overline{} & :: & 7 \textit{ days} \; + \end{array}$$

$$\frac{6 \times 16 \times 24}{9 \times 7}, \textit{ by contraction} = \frac{2 \times 16 \times 24}{3 \times 7} = \frac{2 \times 16 \times 8}{1 \times 7}$$

$$= \frac{256}{7} = 36\tfrac{4}{7} \textit{ horses, the answer.}$$

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* The reason of this rule may be readily shewn from the nature of direct and inverse proportion: for every row in this case is a particular stating in one of those rules; and therefore if all the separate dividends be collected together into one dividend, and all the divisors into one divisor, their quotient must be the answer sought. Thus, in example the first,

As 9 bush. : 16 horses : : 24 bush. : $\frac{24 \times 16}{9}$ by rule of three direct.

As 6 days : $\frac{24 \times 16}{9}$ horses : : 7 days $\frac{24 \times 16 \times 6}{9 \times 7}$ by rule of three inverse, which is the same as the rule.

2. If

2. If a family of 9 people spend 120 *l.* in 8 months, how much will serve a family of 24 people 16 months?

Ans. 640 *l.*

3. If 8 men can dig 24 yards of earth in 6 days, how many men must there be to dig 18 yards in 3 days?

Ans. 12 *men.*

4. If 2 men can do $12\frac{3}{4}$ rods of ditching in $6\frac{1}{2}$ days, how many rods may be done by 18 men in 14 days?

Ans. $247\frac{2}{13}$ *rods.*

5. If a regiment of soldiers, consisting of 939 men, can eat 351 quarters of wheat in 7 months, how many soldiers will eat 1464 quarters in 5 months at that rate?

Ans. $5483\frac{23}{195}$

6. If the carriage of 5 *cwt.* 3 *qr.* 150 miles cost 3 *l.* 7 *s.* 4 *d.* what must be paid for the carriage of 7 *cwt.* 2 *qr.* 25 *lb.* 64 miles, at the same rate? *Ans.* 1 *l.* 18 *s.* 7 *d.* $\frac{117}{2415}$

7. If 248 men, in 5 days, of 11 hours each, can dig a trench 230 yards long, 3 wide, and 2 deep, in how many days, of 9 hours long, will 24 men dig a trench of 420 yards long, 5 wide, and three deep?

Ans. $288\frac{59}{207}$ *days.*

8. If a person travel 300 miles in 10 days, when the day is 12 hours long, in how many days may he travel 600 miles, when the day is 16 hours long?

9. If a barrel of beer be sufficient to last a family of 7 persons 12 days, how many barrels will be drank out by a family of 14 persons in a year?

PRACTICE.

PRACTICE is a compendious method of working the Rule of Three Direct, when the first term is an unit, or one; and is of general use among merchants and tradesmen, on account of its being the most easy and concise manner of answering such questions as commonly occur in business.

An aliquot part of any number is such a part as being taken a certain number of times, will exactly make that number; as $\frac{1}{4}$ is an aliquot part of 1, for being taken 4 times, or multiplied by 4, it produces one; and 2 is an aliquot part of 6, for being taken 3 times, it makes 6, &c.

A TABLE OF ALIQUOT PARTS.

Of a Shilling.

6d.	is	$\frac{1}{2}$	A Half
4d.	is	$\frac{1}{3}$	A Third
3d.	is	$\frac{1}{4}$	A Fourth
2d.	is	$\frac{1}{6}$	A Sixth
$1\frac{1}{2}$d.	is	$\frac{1}{8}$	An Eighth
1d.	is	$\frac{1}{12}$	A Twelfth
$\frac{1}{2}$d.	is	$\frac{1}{2}$	of a Penny
$\frac{1}{4}$d.	is	$\frac{1}{4}$	of ditto

Of a Pound.

10s.	is	$\frac{1}{2}$
6s. 8d.	is	$\frac{1}{3}$
5s.	is	$\frac{1}{4}$
3s. 4d.	is	$\frac{1}{6}$
2s. 6d.	is	$\frac{1}{8}$
2s.	is	$\frac{1}{10}$
1s. 8d.	is	$\frac{1}{12}$
1s.	is	$\frac{1}{20}$

Of an Hundred Weight.

2 Qrs. or 56 lb.	is	$\frac{1}{2}$
1 Qr. or 28 lb.	—	$\frac{1}{4}$
14 lb.	—	$\frac{1}{8}$
7 lb.	—	$\frac{1}{16}$

Of a Quarter of a Cwt.

14 lb.	is	$\frac{1}{2}$
7 lb.	—	$\frac{1}{4}$
4 lb.	—	$\frac{1}{7}$
$3\frac{1}{2}$ lb.	—	$\frac{1}{8}$

CASE I *.

When the price is less than a penny.

RULE.

Divide the given number by the aliquot parts of a penny, and then by 12 and by 20, and it will give the answer required.

EXAM-

* As most of the following compendiums are only particular cases of a more general rule, it will be sufficient, for their illustration, to explain the principles on which the rule itself is founded.

General Rule. 1. Suppose the price of the given quantity to be 1 *l.* or 1 *s.* as is most convenient; then will the quantity itself be the answer at the supposed price.

2. Divide the given price into aliquot parts, either of the supposed price, or of one another, and the sum of the quotients belonging to each, will be the true answer required.

EXAMPLE.

What is the value of 526 yards of cloth, at 3 *s.* $10\frac{1}{4}$ *d. per* yard?

		526			*Ans.* at	1 *l.*		
3 *s.* 4 *d.* is $\frac{1}{6}$		87	13	4	ditto at	0	3	4
4 *d.* is $\frac{1}{10}$		8	15	4	ditto at	0	0	4
2 *d.* is $\frac{1}{2}$		4	7	8	ditto at	0	0	2
$\frac{1}{4}$ is $\frac{1}{8}$		0	10	$11\frac{1}{2}$	ditto at	0	0	$0\frac{1}{4}$
the full price.		101	7	$3\frac{1}{2}$	ditto at	0	3	$10\frac{1}{4}$

In

EXAMPLES.

4506 at $\frac{3}{4}$

$\frac{1}{2}$ is $\frac{1}{2}$ 2253
$\frac{1}{4}$ is $\frac{1}{2}$ $1126\frac{1}{2}$

12)$3379\frac{1}{2}$

2,0)28,1 : 7

14 *l.* 1 *s.* $7\frac{1}{2}$ *d. the anſwer.*

3456 at $\frac{1}{4}$. *Anſ.* 3 *l.* 12 *s.* 347 at $\frac{1}{2}$. *Anſ.* 14 *s.* $5\frac{1}{2}$ *d.*
846 at $\frac{3}{4}$. *Anſ.* 2 *l.* 12 *s.* $10\frac{1}{2}$ *d.* 810 at $\frac{3}{4}$. *Anſ.* 2 *l.* 10 *s.* $7\frac{1}{2}$ *d.*

CASE 2.

When the price is an aliquot part of a ſhilling.

RULE.

Divide the given number by the aliquot part, and the quotient is the anſwer in ſhillings, which reduce into pounds as before.

EXAMPLES.

3 *d.* is $\frac{1}{4}$ 1728 at 3 *d.*

2,0) 43,2

21 *l.* 12 *s.* *the anſwer.*

437 at 1 *d.* *Anſ.* 1 *l.* 16 *s.* 5 *d.* 352 at $1\frac{1}{2}$ *d.* *Anſ.* 2 *l.* 4 *s.*
5275 at 2 *d.* *Anſ.* 43 *l.* 19 *s.* 2 *d.* 1776 at 3 *d.* *Anſ.* 22 *l.* 4 *s.*
6771 at 4 *d.* *Anſ.* 112 *l.* 17 *s.* 899 at 6 *d.* *Anſ.* 22 *l.* 9 *s.* 6 *d.*

In the above example, it is plain, that the quantity 526, is the anſwer at 1 *l.* conſequently, as 3 *s.* 4 *d.* is the $\frac{1}{6}$ of a pound, $\frac{1}{6}$ part of that quantity, or 87 *l.* 13 *s.* 4 *d.* is the price at 3 *s.* 4 *d.* In like manner, as 4 *d.* is the $\frac{1}{10}$ part of 3 *s.* 4 *d.* ſo $\frac{1}{10}$ of 87 *l.* 13 *s.* 4 *d.* or 8 *l.* 15 *s.* 4 *d.* is the anſwer at 4 *d.* And by reaſoning in this way 4 *l.* 7 *s.* 8 *d* will appear to be the price at 2 *d.* and 10 *s.* $11\frac{1}{2}$ *d.* the price at $\frac{1}{4}$. Now as the ſum of all theſe parts is equal to the whole price, (3 *s.* $10\frac{1}{4}$ *d.*) ſo the ſum of the anſwers belonging to each price will be the anſwer at the full price required: And the ſame will be true of any example whatever.

CASE 3.

When the price is pence and farthings, and is no aliquot part of a ſhilling.

RULE.

Find what aliquot part of a ſhilling is neareſt to the given price, and divide the propoſed number by it:

And if there be any remainder, conſider what part it is of this aliquot part of the given price, and divide the former quotient by it, &c. and the ſeveral quotients, added together, will be the anſwer in ſhillings, which reduce into pounds as before.

EXAMPLES.

	876	*at* $8\frac{1}{2}$ *d.*
6 *d. is* $\frac{1}{2}$	438	
2 *d. is* $\frac{1}{3}$	146	
$\frac{1}{2}$ *d. is* $\frac{1}{4}$	36	- 6
2,0)	62,0	- 6
	31 *l.* - 0 *s.* 6 *d.*	*the anſwer.*

372 at $1\frac{3}{4}$ *d.*	*Anſ.*	2 *l.*	14 *s.*	3	*d.*
325 at $2\frac{1}{2}$ *d.*	*Anſ.*	3 *l.*	0 *s.*	$11\frac{1}{4}$	*d.*
827 at $4\frac{1}{2}$ *d.*	*Anſ.*	15 *l.*	10 *s.*	$1\frac{1}{2}$	*d.*
2700 at $7\frac{1}{4}$ *d.*	*Anſ.*	81 *l.*	11 *s.*	3	*d.*
2150 at $9\frac{3}{4}$ *d.*	*Anſ.*	87 *l.*	6 *s.*	$10\frac{1}{2}$	*d.*
1720 at $11\frac{1}{2}$ *d.*	*Anſ.*	82 *l.*	8 *s.*	4	*d.*

CASE 4.

When the price is any number of ſhillings under 20.

RULE.

1. When the price is an even number, multiply the given number by $\frac{1}{2}$ of it, doubling the firſt figure to the right-hand for ſhillings, and the reſt are pounds.

2. When the price is an odd number, find for the greateſt even number as before, to which add $\frac{1}{20}$ of the given number for the odd ſhilling, and the ſum is the anſwer.

EXAM-

EXAMPLES.

	243	at 4 *s.*	
	2		
	48 *l.*	12 *s.*	*the answer.*
	566	at 7 *s.*	
	3		
	169 -	16	
1 *s.* is $\frac{1}{20}$	28 -	6	
	198 *l.* -	2 *s.*	*the answer.*

2757 at 1 *s.*	*Ans.* 137 *l.*	17 *s.*
2643 at 2 *s.*	*Ans.* 264 *l.*	6 *s.*
3271 at 5 *s.*	*Ans.* 817 *l.*	15 *s.*
872 at 8 *s.*	*Ans.* 348 *l.*	16 *s.*
372 at 11 *s.*	*Ans* 204 *l.*	12 *s.*
5271 at 14 *s.*	*Ans.* 3689 *l.*	14 *s.*
3142 at 17 *s.*	*Ans.* 2670 *l*	14 *s.*
264 at 19 *s.*	*Ans.* 250 *l.*	16 *s.*

CASE 5.

When the price is shillings and pence, which make some aliquot part of a pound.

RULE.

Divide the given quantity by the aliquot part, and the quotient is the answer in pounds.

EXAMPLES.

3 *s.* 4 *d.* is $\frac{1}{6}$	329	at 3 *s.* 4 *d.*	
	54 *l.*	16 *s.* 8 *d.*	*the answer.*

7150 at 1 *s.* 8 *d.*	*Ans.* 595 *l.*	16 *s.* 8 *d.*
2715 at 2 *s.* 6 *d.*	*Ans* 339 *l.*	7 *s.* 6 *d.*
3150 at 3 *s.* 4 *d.*	*Ans.* 525 *l.*	0 *s* 0 *d.*
2710 at 6 *s.* 8 *d.*	*Ans.* 903 *l.*	6 *s.* 8 *d.*

CASE 6.

When the price is ſhillings and pence, which make no aliquot part of a pound.

RULE.

Take the neareſt leſs ſum, which is an aliquot part, and find the value of the quantity propoſed at that rate; then to this ſum add the amount of the remaining parts of the price, found by ſome of the foregoing rules, and it will give the anſwer required.

EXAMPLES.

765 *at* 5 *s.* 9 *d.*

5 *s.* *is* $\frac{1}{4}$	191	- 5	
6 *d.* *is* $\frac{1}{10}$	19	- 2	- 6
3 *d.* *is* $\frac{1}{2}$	9	- 11	- 3

219 *l.* - 18 *s.* - 9 *d.* *the anſwer.*

7211	at	1 *s.* 3 *d.*	*Anſ.*	450 *l.* 13 *s.* 9 *d.*
2701	at	3 *s.* 2 *d.*	*Anſ.*	421 *l.* 1 *s.* 2 *d.*
2547	at	7 *s.* 3 *d.*	*Anſ.*	923 *l.* 5 *s.* 9 *d.*
801	at	10 *s.* 9 *d.*	*Anſ.*	430 *l.* 10 *s.* 9 *d.*
841	at	13 *s.* 2 *d.*	*Anſ.*	553 *l.* 13 *s.* 2 *d.*
807	at	16 *s.* 5 *d.*	*Anſ.*	662 *l.* 8 *s.* 3 *d.*
309	at	17 *s.* 3 *d.*	*Anſ.*	266 *l.* 10 *s.* 3 *d.*
846	at	18 *s.* 6 *d.*	*Anſ.*	782 *l.* 11 *s.*
969	at	19 *s.* 11 *d.*	*Anſ.*	964 *l.* 19 *s.* 3 *d.*

CASE 7.

When the price is ſhillings, pence and farthings.

RULE.

Divide the price into aliquot parts of a pound, or of each other, and the ſum of the quotients, belonging to each aliquot part, will be the anſwer required.

EXAM-

EXAMPLES.

244 at 5 *s.* 8½ *d.*

5 *s. is* ¼	61					
6 *d. is* $\frac{1}{10}$	6	-	2			
2 *d. is* ⅓	2	-	0	-	8	
½ *d. is* ¼	-	-	10	-	2	

69 *l.* - 12 *s.* - 10 *d.* *the answer.*

875 at	1 *s.*	4¾ *d.*	*Ans.*	61 *l.*	1 *s.*	4¼ *d.*
7524 at	3 *s.*	5½ *d.*	*Ans.*	1332 *l.*	7 *s.*	6 *d.*
3715 at	9 *s.*	4½ *d.*	*Ans.*	1741 *l.*	8 *s.*	1½ *d.*
2572 at	13 *s.*	7½ *d.*	*Ans.*	1752 *l.*	3 *s.*	6 *d.*
1603 at	16 *s.*	10½ *d.*	*Ans.*	1352 *l.*	10 *s.*	7½ *d.*
2710 at	19 *s.*	2½ *d.*	*Ans.*	2602 *l.*	14 *s.*	7 *d.*
430 at	19 *s.*	6¼ *d.*	*Ans.*	419 *l.*	13 *s.*	11½ *d.*

CASE 8.

When the price is pounds, ſhillings, pence and farthings.

RULE.

Multiply the quantity propoſed by the number of pounds, and work for the reſt by ſome of the former rules; and theſe ſums added together, will give the anſwer required.

EXAMPLES.

428 at 3 *l.* 4 *s.* 6½ *d.*

	428					
	3					
	1284					
4 *s. is* ⅕	85	-	12			
6 *d. is* ⅛	10	-	14			
½ *d. is* $\frac{1}{12}$	—	-	17	-	10	

1381 *l.* - 3 *s.* - 10 *d.* *the answer.*

137 at	1 *l.*	17 *s.*	6¼ *d.*	*Ans.*	257 *l.*	0 *s.*	4½ *d.*
947 at	4 *l.*	15 *s.*	10¼ *d.*	*Ans.*	4538 *l.*	13 *s.*	10¼ *d.*
457 at	14 *l.*	17 *s.*	9½ *d.*	*Ans.*	6804 *l.*	10 *s.*	9½ *d.*
713 at	19 *l.*	19 *s.*	11¾ *d.*	*Ans.*	14259 *l.*	5 *s.*	1¾ *d.*

CASE

CASE 9.

When the number whose price is required is a whole number, with parts annexed.

RULE.

Work for the whole number according to the former rules, to which add $\frac{1}{4}$, $\frac{1}{2}$ or $\frac{3}{4}$ of the price, according as the question requires.

EXAMPLES.

$234\frac{3}{4}$ *at* 5 *s.* 8 *d.*

5 *s.* *is* $\frac{1}{4}$	58	-	10
6 *d.* *is* $\frac{1}{10}$	5	-	17
2 *d.* *is* $\frac{1}{3}$	1	-	19
for $\frac{1}{2}$	2	-	10
for $\frac{1}{4}$	1	-	5
	70 *l.*	-	1 *s.* *the answer.*

$273\frac{1}{4}$ at	2 *s.* 6 *d.*	*Ans.*	34 *l.*	3 *s.*	$1\frac{1}{2}$ *d.*
$937\frac{1}{2}$ at	3 *l.* 17 *s.* 8 *d.*	*Ans.*	3640 *l.*	12 *s.*	6 *d.*
$139\frac{3}{4}$ at	1 *l.* 19 *s.* 4 *d.*	*Ans.*	274 *l.*	16 *s.*	10 *d.*
$371\frac{3}{4}$ at	4 *l.* 13 *s.* 7 *d.*	*Ans.*	1739 *l.*	9 *s.*	$7\frac{1}{4}$ *d.*
$284\frac{1}{2}$ at	2 *l.* 10 *s.* 6 *d.*	*Ans.*	718 *l.*	7 *s.*	3 *d.*

CASE 10.

When the quantity whose value is required is of several denominations.

RULE.

Find the value of the highest denomination, by some of the foregoing rules; and for the others take such parts of the given price as the lower denominations are of the higher, or of each other, as is most convenient; and the several sums added together, will give the answer required.

EXAM-

EXAMPLES.

8 *cwt.* 2 *qr.* 16 *lb. at* 2 *l.* 5 *s.* 6 *d.*

	2*l.* 5*s.* 6*d.*
	8
	18 - 4 - -
2 *qr. is* $\frac{1}{2}$	1 - 2 - 9
14 *lb. is* $\frac{1}{4}$	- - 5 - 8$\frac{1}{4}$
2 *lb. is* $\frac{1}{7}$	- - - - 9$\frac{3}{4}$
	19*l.* 13*s.* 3*d. the answer.*

37 *cwt.* 2 *qrs.* 14 *lb.* at 7 *l.* 10 *s.* 9 *d. per cwt.*
Ans. 283 *l.* 11 *s.* 11$\frac{1}{2}$ *d.*

17 *cwt.* 1 *qr.* 12 *lb.* at 1 *l.* 19 *s.* 8 *d. per cwt.*
Ans. 34 *l.* 8 *s.* 5$\frac{1}{4}$ *d.*

23 *cwt.* 3 *qrs.* 8 *lb.* at 3 *l.* 19 *s.* 11 *d. per cwt.*
Ans. 95 *l.* 3 *s.* 8$\frac{3}{4}$ *d.*

39 *cwt.* 0 *qr.* 10 *lb.* at 1 *l.* 17 *s.* 10 *d. per cwt.*
Ans. 73 *l.* 18 *s.* 10$\frac{1}{4}$ *d.*

PROMISCUOUS QUESTIONS.

73 *cwt.* 1 *qr.* of sugar, at 3 *l.* 15 *s.* 7 *d. per cwt.?*
17 *tons,* 2 *cwt.* 3 *qrs.* 12 *lb.* at 9 *l. per ton.?*
3 *qrs.* 12$\frac{1}{2}$ *lb.* at 2 *l.* 16 *s.* 10 *d. per cwt.?*
24 *sacks,* 9 *tods,* 1 *stone* of wool at 2 *l.* 10 *s.* 6 *d. per sack?*
125 *yards,* 3 *qrs.* of cloth, at 2 *s.* 8$\frac{1}{2}$ *d. per yard?*
13 *eng. ells,* 2 *qrs.* 2 *nls.* at 3 *s.* 7$\frac{1}{2}$ *d. per ell?*
713 *acres,* 3 *roods,* 39 *pls.* at 3 *l.* 17 *s.* 6 *d. per acre?*
75 *hhds.* 1 *tierce* of wine, at 25 *l.* 13 *s.* 6 *d. per hhd?*
24 *gals.* 3 *qrs.* of oil, at 3 *s.* 4$\frac{1}{2}$ *per gallon?*
57 *hhds.* 41 *gals.* of ale, at 2 *l.* 10 *s.* 6 *d. per hhd?*
43 *qrs.* 5 *bus.* of wheat, at 1 *l.* 8 *s.* 6 *d. per quarter?*
What is the hire of a coach and horses, for 9 *months* and 11 *days,* at 5 *l.* 10 *s. per month?*

TARE AND TRETT.

TARE and TRETT are practical rules for deducting certain allowances, which are made by merchants and tradesmen in selling their goods by weight.

TARE,

TARE, is an allowance to the buyer for the weight of the box, barrel, bag, &c. which contains the goods bought, and is either at ſo much *per* box, &c. at ſo much *per cwt.* or at ſo much in the groſs weight.

TRETT, is an allowance of 4 *lb.* in every 104 *lb.* for waſte, duſt, &c.

CLOFF, is an allowance, after tare and trett are deducted, of 2 *lb.* upon every 3 *cwt.*

GROSS WEIGHT, is the whole weight of the goods, together with the box, barrel, bag, &c. that contains them.

SUTTLE is when only part of the allowance is deducted from the groſs.

NEAT WEIGHT is what remains after all allowances are made.

CASE I.

When the tare is at ſo much *per* box, barrel, or bag, &c.

RULE*.

Multiply the number of boxes, barrels, &c. by the tare, and ſubtract the product from the groſs, and the remainder will be the neat weight required.

EXAMPLES.

1. In 7 frails of raiſins, each weighing 5 *cwt.* 2 *qrs.* 5 *lb.* groſs, tare 23 *lb. per.* frail, how much neat?

23 × 7 = 1 *cwt.* 1 *qr.* 21 *lb.*

cwt.	*qr.*	*lb.*	
5	2	5	
		7	
38	3	7	*groſs*
1	1	21	*tare*

37 *cwt.* 1 *qr.* 14 *lb.* *the anſwer.*

2. In 241 barrels of figs, each 0 *cwt.* 3 *qr.* 19 *lb.* groſs, tare 10 *lb. per* barrel, how many pounds neat?

Anſ. 22413 *lb.*

3. What is the neat weight of 14 *hhds.* of tobacco, each 5 *cwt.* 2 *qrs.* 17 *lb.* groſs, tare 100 *lb. per hhd.*?

Anſ. 66 *cwt.* 2 *qr.* 14 *lb.*

* It is manifeſt, that this, as well as every other caſe in this rule, is only an application of the rules of proportion and practice.

4. What

4. What is the neat weight of 17 bags of cotton yarn, each weighing 2 *cwt.* 3 *qrs.* 4 *lb.* groſs, tare 9 *lb.* *per* bag?

Anſ. 45 *cwt.* 3 *qr.* 27 *lb.*

CASE 2.

When the tare is at ſo much *per* cwt.

RULE.

Divide the groſs weight by the aliquot parts of a *cwt.* and ſubtract the ſum of the quotients from the groſs, and the remainder will be the neat weight required.

EXAMPLES.

1. Groſs 173 *cwt.* 3 *qr.* 17 *lb.* tare 16 *lb.* *per cwt.* how much neat?

	cwt.	*qr.*	*lb.*	
	173	3	17	*groſs*
14 *lb.* *is* $\frac{1}{8}$	21	2	26	
2 *lb.* *is* $\frac{1}{7}$	3	0	11	
	24	3	9	
	149	0	8	*the anſwer.*

2. What is the neat weight of 7 barrels of pot-aſh, each weighing 201 *lb.* groſs, tare being at 10 *lb.* *per cwt.*?

Anſ. 1281 *lb.* 6 *oz.*

3. In 25 barrels of figs, each 2 *cwt.* 1 *qr.* groſs, tare 16 *lb.* *per cwt.* how much neat? *Anſ.* 48 *cwt.* 0 *qr.* 24 *lb.*

4. What is the value of the neat weight of 13 *hhds.* of tobacco, at 4 *l.* 13 *s.* 6 *d.* *per cwt.* each weighing 4 *cwt.* 3 *qr.* 17 *lb.* groſs, tare 13 *lb.* *per cwt.*

Anſ. 263 *l.* 6 *s.* $\frac{1}{2}$ *d.*

CASE 3.

When there is an allowance both of tare and trett.

RULE.

Subtract the tare from the groſs weight, by the foregoing rules, and the remainder, or ſuttle, divided by 26, gives the trett, which being ſubtracted from the ſuttle, leaves the neat weight required.

EXAMPLES.

1. In 9 *cwt.* 2 *qr.* 17 *lb.* groſs, tare 37 *lb.* and trett as uſual, how much neat?

cwt.	*qr.*	*lb.*	
9	2	17	*groſs*
0	1	9	*tare*
26)9	1	8	*ſuttle*
	1	12	*trett*
8	3	24	*the anſwer.*

2. In 152 *cwt.* 1 *qr.* 3 *lb.* groſs, tare 10 *lb. per cwt.* and trett as uſual, how much neat?

Anſ. 133 *cwt.* 1 *qr.* 12 *lb.*

3. In 7 caſks of prunes, each weighing 3 *cwt.* 1 *qr.* 5 *lb.* groſs, tare $17\frac{1}{2}$ *lb. per cwt.* and trett as uſual, how much neat? *Anſ.* 18 *cwt.* 2 *qr.* 25 *lb.*

4. What is the neat weight of 3 *hhds.* of ſugar, weighing as follows: the 1ſt. 4 *cwt.* 0 *qr.* 5 *lb.* groſs, tare 73 *lb.*; the 2d. 3 *cwt.* 2 *qr.* groſs, tare 56 *lb.*; and the 3d. 2 *cwt.* 3 *qr.* 17 *lb.* groſs, tare 47 *lb.* and allowing trett to each as uſual?

Anſ. 8 *cwt.* 2 *qr.* 4 *lb.*

CASE 4.

When tare, trett, and cloff are all allowed.

RULE.

Deduct the tare and trett, as before, and divide the remainder, or ſuttle, by 168, and the quotient is the cloff, which being ſubtracted from the ſuttle, the remainder is the neat weight.

EXAMPLES.

What is the neat weight of a *hhd.* of tobacco, weighing 15 *cwt.* 3 *qr.* 20 *lb.* groſs, tare 7 *lb. per cwt.* and trett and cloff as uſual.

cwt.

	cwt.	qr.	lb.	
	15	3	20	*gross*
7 *lb.* is $\frac{1}{16}$	—	3	27	*tare*
26)	14	3	21	
	—	2	8	*trett*
168)	14	1	13	*suttle*
	—	-	9	*cloff*
	14	1	4	*the answer.*

2. In 19 chests of sugar, each containing 13 *cwt.* 1 *qr.* 17 *lb.* gross, tare 13 *lb. per cwt.* and trett and cloff as usual, how much neat, and what is the value at $5\frac{1}{4}$ *d. per lb.*?

Ans. 215 *cwt.* 0 *qr.* 17 *lb. and value* 577 *l.* 6 *s.* $5\frac{3}{4}$ *d.*

3. 29 parcels, each weighing 3 *cwt.* 0 *qr.* 14 *lb.* gross; what is the value of the neat weight at 1 *l.* 11 *s.* 6 *d. per cwt.* allowing 8 *lb. per cwt.* for tare, and trett and cloff as usual?

Ans. 126 *l.* 14 *s.* $0\frac{3}{4}$ *d.*

4. What is the value of the neat weight of 5 *hhds.* of tobacco, each weighing 5 *cwt.* 2 *qrs.* 25 *lb.* gross, at 8 *l.* 12 *s.* 6 *d. per cwt.* allowing 8 *lb. per cwt.* for tare, trett as usual, and cloff 2 *lb. per hhd.*?

BILLS OF PARCELS.

A HOSIER'S BILL.

Mr. Thomas Williams

Bought of Richard Simpson, Jan. 4, 1786.

		s.	d.	
8 Pair of worsted stockings,	at	4	6	*per pair.*
5 Pair of thread ditto,	at	3	2	
3 Pair of black silk ditto,	at	14	0	
6 Pair of black worsted ditto,	at	4	2	
4 Pair of cotton ditto,	at	7	6	
2 Yards of fine flannel,	at	1	8	*per yard.*

£. 7 12 2

A MERCER'S BILL.

Mr. William George

Bought of Peter Thompson, July 13, 1786.

			s.	d.	
15	Yards of sattin,	at	9	6	*per yard.*
18	Yards of flowered silk,	at	17	4	
12	Yards of rich brocade,	at	19	8	
16	Yards of sarsnet,	at	3	2	
13	Yards of Genoa velvet,	at	27	6	
23	Yards of luteftring,	at	6	3	

£. 62 2 5

A LINEN-DRAPER'S BILL.

Mr. Henry Morris

Bought of Caleb Windsor, March 8, 1786.

			s.	d.	
40	Ells of dowlas,	at	1	6	*per ell.*
34	Ells of diaper,	at	1	$4\frac{1}{2}$	
31	Ells of Holland,	at	5	8	
39	Yards of Irish cloth,	at	2	4	*per yard.*
$17\frac{1}{2}$	Yards of muslin,	at	7	$2\frac{1}{2}$	
$13\frac{3}{4}$	Yards of cambric,	at	10	6	
27	Yards of printed linen,	at	2	5	

£. 35 9 $2\frac{1}{4}$

A MILLINER'S BILL.

Mrs. Matthewson

Bought of Simon Percy, June 18, 1786.

			l.	s.	d.	
18	Yards of fine lace,	at	0	12	3	*per yard.*
5	Pair of fine kid gloves,	at	0	2	2	*per pair.*
12	Fans with French mounts,	at	0	3	6	*each.*
2	Fine laced tippets,	at	3	3	0	
4	Dozen of linen gloves,	at	0	1	3	*per pair.*
6	Sets of knots,	at	0	2	6	*per set.*

£. 23 14 4

A WOOL-

A WOOLLEN-DRAPER'S BILL.

Mr. John Page

Bought of Jacob Goodson, May 1, 1786.

		l.	s.	d.	
17 Yards of fine serge,	at	0	3	9	*per yard.*
18 Yards of drugget,	at	0	9	0	
15 Yards of superfine scarlet,	at	1	2	0	
16 Yards of super. black cloth,	at	0	18	0	
25 Yards of shalloon,	at	0	1	9	
17 Yards of drab,	at	0	17	6	
		£. 59	5	0	

A GROCER'S BILL.

Mr. Nathaniel Parsons

Bought of William Smith, Aug. 6, 1786.

		s.	d.	
$24\frac{1}{2}$ *lb.* of royal green tea,	at	18	6	*per lb.*
$24\frac{1}{4}$ *lb.* of imperial tea,	at	24	0	
$35\frac{3}{4}$ *lb.* of best bohea,	at	13	10	
17 *lb.* of coffee,	at	5	4	
25 *lb.* of double refined sugar,	at	1	$1\frac{1}{2}$	
9 Sugar loaves, wt. 137 *lb.*	at	0	$7\frac{1}{2}$	
		£. 86	14	$2\frac{1}{2}$

A WINE-MERCHANT'S BILL.

Mr. Thomas Greville

Bought of John Simes, April 3, 1785.

		s.	d.	
12 Gallons of palm sack,	at	8	6	*per gall.*
17 Gallons of red port,	at	5	8	
9 Gallons of claret,	at	8	9	
34 Gallons of white Lisbon,	at	4	10	
$22\frac{1}{2}$ Gallons of rhenish,	at	6	4	
$27\frac{3}{4}$ Gallons of sherry,	at	6	2	
		£. 37	15	$0\frac{1}{2}$

A CHEESE-MONGER'S BILL.

Mr. Edward Patterson

Bought of Stephen Cross, Sept. 1, 1786.

		s.	d.
8 *lb* of Cambridge butter,	at	0	6 *per lb.*
17 *lb.* of new cheese,	at	0	4
$\frac{1}{2}$ Firkin of butter, wt. 28 *lb.*	at	0	$5\frac{1}{2}$
5 Cheshire cheeses, wt. 127 *lb.*	at	0	4
2 Warwickshire ditto, wt. 15 *lb.*	at	0	3
2 *lb.* of cream cheese,	at	0	6

£. 3 9 7

SIMPLE INTEREST.

SIMPLE INTEREST is an allowance made by the borrower of any sum of money to the lender, according to a certain rate *per annum*; which, by law, must not exceed 5 *per cent.* that is, 5*l.* for the use of 100*l.* 1 year; 10*l.* for the use of it 2 years; and so on.

PRINCIPAL is the money lent.

RATE is the sum *per cent.* agreed on.

AMOUNT is the principal and interest added together.

RULE*.

1. Multiply the principal by the rate, and divide the product by 100, and the quotient is the interest for 1 year.

2. Multiply the interest for 1 year by the number of years given, and the product is the interest for that time.

3. If parts of a year, as months or days, be given, they must be worked for by the aliquot parts of a year, or by the rule of three direct.

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* There are some cases where it is customary to consider the time elapsed different ways. In the courts of law, interest is always computed in years, quarters and days; which, indeed, is the only equitable method; but in computing the interest on the public bonds of the South-Sea and India companies, and in the Bank of England, &c. the time is generally taken in calendar months and days; and on Exchequer bills in quarters of a year and days.

If the interest be found, according to either of these methods, and then added to the principal, it will give the amount, or whole sum which is due.

EXAM-

EXAMPLES.

1. What is the intereſt of 284 *l.* 10 *s.* for 2 years, 4 months, and 25 days, at $3\frac{1}{2}$ *per cent. per annum?*

284 *l.*	10 *s.*	365 : 9 *l.*	19 *s.*	$1\frac{3}{4}$ *d.*	:: 25 *days*	
	$3\frac{1}{2}$			5		
853	10	49	15	$8\frac{3}{4}$		
142	5			5		
9.95	15	365)248	18	$7\frac{3}{4}$	(13 *s.* $7\frac{1}{2}$ *d.*	
20		20				
19.15		4978				
12		1328				
		233				
1.80		12				
4						
		2803				
3.20		248				
		4				
		995				
		265				

	l.	*s.*	*d.*		
	9 *l.*	19 *s.*	$1\frac{3}{4}$ *d.*	=	1 *year's intereſt.*
			2		
	19	18	$3\frac{1}{2}$	=	2 *year's intereſt.*
4 *mo.* = $\frac{1}{3}$	3	6	$4\frac{1}{2}$	=	4 *month's ditto.*
		13	$7\frac{1}{2}$	=	25 *day's ditto.*
	23	18	$3\frac{1}{2}$		*the anſwer required.*

2. What is the intereſt of 230 *l.* 10 *s.* for 1 year at 4 *per cent. per annum?* *Anſ.* 9 *l.* 4 *s.* $4\frac{3}{4}$ *d.*

3. What is the intereſt of 547 *l.* 15 *s.* for 3 years, at 5 *per cent. per annum?* *Anſ.* 82 *l.* 3 *s.* 3 *d.*

4. What is the amount of 690 *l.* for three years, at $4\frac{1}{2}$ *per cent. per annum?* *Anſ.* 783 *l.* 3 *s.*

5. What is the intereſt of 205 *l.* 15 *s.* for $\frac{1}{4}$ year, at 4 *per cent. per annum?* *Anſ.* 2 *l.* 1 *s.* $1\frac{3}{4}$ *d.*

6. What is the amount of 120 *l.* 10 *s.* for $2\frac{1}{2}$ years, at $4\frac{3}{4}$ *per cent. per annum?* *Anſ.* 134 *l.* 16 *s.* $1\frac{3}{4}$ *d.*

7. What is the interest of 47 *l.* 10 *s.* for 4 years, and 52 days, at $4\frac{1}{2}$ *per cent.?* *Ans.* 10 *l.* 9 *s.* $1\frac{3}{4}$ *d.*

8. What is the amount of 200 guineas for 4 years, 7 months and 25 days, at $4\frac{1}{2}$ *per cent.?* *Ans.* 253 *l.* 19 *s.* $2\frac{1}{4}$ *d.*

9. A gentleman left his niece by will 558 *l.* 15 *s.* to be paid her when she came to age, with interest at 4 *per cent.* now she came to age in 5 years, 9 months and 21 days; what has she to receive in all? *Ans.* 688 *l.* 10 *s* $11\frac{1}{2}$ *d.*

10. What is the interest due upon an India bond of 500 *l.* value, at $3\frac{1}{2}$ *per cent. per ann.* from September 30, 1786, to June 18, 1787? *Ans.* 12 *l.* 10 *s.* $3\frac{3}{4}$ *d.*

11. What is the interest due upon an Exchequer bill of 450 *l.* at $3\frac{3}{4}$ *per cent. per ann.* for $2\frac{3}{4}$ *years,* and 67 *days?* *Ans.* 49 *l.* 10 *s.* $0\frac{3}{4}$ *d.*

COMMISSION*.

COMMISSION is an allowance of so much *per cent.* to a factor or correspondent abroad for buying and selling goods for his employer.

EXAMPLES.

1. What comes the commission of 500 *l.* 13 *s.* 6 *d.* to at $3\frac{1}{2}$ *per cent.?*

£.	s.	d.
500 -	13 -	6
		$3\frac{1}{2}$
1502 -	0 -	6
250 -	6 -	9
17.52 -	7 -	3
20		
10.47		
12		
5.67		
4		
2.68		

Ans. 17 *l.* 10 *s.* $5\frac{1}{2}$ *d.*

* The method of working questions in this and the following rules of insurance, brokerage, &c. is the same as in simple interest.

2. My

2. My correspondent writes me word thas he has bought goods on my account to the value of 754 *l.* 16 *s.* what does his commission come to at $2\frac{1}{2}$ *per cent.?*

Ans. 18 *l.* 17 *s.* $4\frac{3}{4}$ *d.*

3. What must I allow my correspondent for disbursing on my account 529 *l.* 18 *s.* 5 *d.* at $2\frac{1}{4}$ *per cent.?*

Ans. 11 *l.* 18 *s.* $5\frac{1}{2}$ *d.*

4. If I allow my factor $7\frac{5}{8}$ *per cent* for commission, what may he demand on the laying out 1200 *l.?* *Ans.* 91 *l.* 10 *s.*

5. What does the commission on 950 *l.* come to at $3\frac{7}{8}$ *per cent.?* *Ans.* 36 *l.* 16 *s.* 3 *d.*

BROKERAGE.

BROKERAGE is an allowance of so much *per cent.* to a person called a broker, for assisting merchants or factors in procuring or disposing of goods.

EXAMPLES.

1. What is the brokerage of 610 *l.* at 5 *s.* or $\frac{1}{4}$ *per cent.?*

```
                £.
   5 s. is 1/4  610
               ------
               1.52 - 10
                 20
               ------
              10.50
                 12
               ------
               6 00
```

Ans. 1 *l.* 10 *s.* 6 *d.*

2. If I allow my broker $3\frac{3}{4}$ *per cent.* what may he demand when he sells goods to the value of 876 *l.* 5 *s.* 10 *d.?*

Ans. 32 *l.* 17 *s.* $2\frac{1}{2}$ *d.*

3. What is the brokerage of 879 *l.* 18 *s.* at $\frac{3}{8}$ *per cent.?*

Ans. 3 *l.* 5 *s.* $11\frac{3}{4}$ *d.*

4. If a broker sells goods to the amount of 508 *l.* 17 *s.* 10 *d.* what is his demand at $1\frac{1}{2}$ *per cent.?* *Ans.* 7 *l.* 12 *s.* 8 *d.*

5. What is the brokerage of 1087 *l.* 15 *s.* $6\frac{1}{2}$ *d.* at $1\frac{5}{8}$ *per cent.?*

6. If a broker sells goods to the amount of 1000 guineas, what is his demand at $\frac{5}{8}$ *per cent.?*

7. If I allow a broker $1\frac{1}{4}$ *per cent.* what is his demand for disposing of goods to the value of 729 *l.* 10 *s.* 6 *d.?*

INSU-

INSURANCE.

INSURANCE is an allowance of so much *per cent.* given to certain persons and offices who engage to make good the loss of ships, houses, or merchandizes, which may happen from storms, fire, &c.

EXAMPLES.

1. What is the insurance of 874 *l.* 14 *s.* 2 *d.* at $12\frac{1}{2}$ *per cent.*?

			£.	s.	d.
			874	14	2
10	*is*	$\frac{1}{10}$	87	9	5
2	*is*	$\frac{1}{5}$	17	9	$10\frac{1}{2}$
$\frac{1}{2}$	*is*	$\frac{1}{2}$	8	14	$11\frac{1}{4}$
			113	14	$2\frac{3}{4}$ *Ans.*

2. What is the insurance of 900 *l.* at $10\frac{3}{4}$ *per cent.*?
Ans. 96 *l.* 15 *s.*
3. What is the insurance of 1200 *l.* at $7\frac{5}{8}$ *per cent.*?
Ans. 91 *l.* 10 *s.*
4. What is the insurance of an East-India ship and cargo valued at 35727 *l.* 17 *s.* 6 *d.* at $17\frac{7}{8}$ *per cent.*?
Ans. 6386 *l.* 7 *s.* $1\frac{3}{4}$ *d.*

BUYING AND SELLING OF STOCKS.

STOCK is a general name for the capitals of our trading companies, and the buying and selling certain sums of money in those funds is now become a general practice.

EXAMPLES.

1. What is the purchase of 2054 *l.* 16 *s.* South-Sea stock, at $110\frac{1}{4}$ *per cent.*?

			£.	s.	
10	*is*	$\frac{1}{10}$	2054	16	
			205	9	7
$\frac{1}{4}$	*is*	$\frac{1}{40}$	5	2	$8\frac{1}{4}$
			2265 *l.*	8 *s.*	$3\frac{3}{4}$ *the answer.*

2. What

2. What is the purchase of 156 *l.* 15 *s.* 3 *per cent.* annuities, at $74\frac{1}{2}$ *per cent.*? *Ans.* 116 *l.* 15 *s.* $6\frac{3}{4}$ *d.*

3. What is the purchase of 816 *l.* 12 *s.* bank annuities, at $89\frac{3}{8}$ *per cent.*? *Ans.* 729 *l.* 16 *s.* $8\frac{1}{2}$ *d.*

4. What is the purchase of 987 *l.* 15 *s.* India stock, at $113\frac{7}{8}$ *per cent.*? *Ans.* 1124 *l.* 16 *s.*

5. Bought 650 *l.* Bank annuities, at $90\frac{3}{8}$ *per cent.* and paid brokerage $\frac{1}{8}$ *per cent.* what did the whole amount to? *Ans.* 588 *l.* 5 *s.*

6. What does 2400 *l.* capital stock in the 3 *per cent.* consolidated bank annuities come to, at $84\frac{1}{8}$ *per cent.*? *Ans.* 2019 *l.*

DISCOUNT.

DISCOUNT is an allowance made for the payment of any sum of money before it becomes due, according to a certain rate *per cent.* agreed on between the parties concerned.

The *present worth* of any sum, or debt, due some time hence, is such a sum as, if put to interest, for that time, at a certain rate *per cent.* would amount to the sum or debt then due.

RULE*.

1. As the amount of 100 *l.* for the given rate and time, is to 100 *l.*

So is the given sum, or debt, to the present worth.

2. Subtract the present worth from the given sum, and the remainder is the discount required.

Or,

As the amount of 100 *l.* for the given rate and time, is to the interest of 100 *l.* for that time,

So is the given sum, or debt, to the discount required.

EXAM-

* That an allowance ought to be made for paying money before it becomes due, which is supposed to bear no interest till after it is due, is highly reasonable; for if I keep the money in my own hands till the debt becomes due, it is plain I may make an advantage of it by putting it out to interest for that time: but if I pay it before it is due, it is giving that benefit to another; therefore we have only to enquire what discount ought to be allowed. And here some debtors may say, that since, by not paying the money till it becomes due, they may employ it at interest; therefore by paying it before it is due, they shall lose that advantage, and, for that reason,

EXAMPLES.

1. What is the diſcount of 573 *l.* 15 *s.* due 3 years hence, at $4\frac{1}{2}$ *per cent.*?

```
                          £.   s.
                           4 - 10
                                3
                          -------
                          13 - 10
 £.    s.                 £.   s.          £.    s.
113 - 10        :         13 - 10    ::   573 - 15
 20                       20               20
-----                    -----            -----
2270                      270              11475
                        11475
                       ------
                       803250
                      22950
                     --------- (20)
             227,0)309825,0(136,4
                     828       ------
                     1472      68 - 4
                      1105
                       197
                        12
                      -----
                 227)2364(10
                       94
                        4
                      ----
                 227)376(¼
                      149
```

Anſ. 68 *l.* 4 *s.* 10¼ *d.*

2. What

reaſon, all ſuch intereſt ought to be diſcounted: but this reaſoning is falſe, for they cannot be ſaid to loſe that intereſt till the time the debt becomes due arrives; whereas we are to conſider what would properly be loſt at preſent, by paying the debt before it becomes due; and this can, in point of equity or juſtice, be no other than ſuch a ſum, as being put out to intereſt till the debt becomes due, would amount to the intereſt of the debt for that time.—It is, beſides, plain, that the advantage ariſing from diſcharging a debt, due ſome time hence, by a preſent payment, according to the principles we have mentioned, is exactly the ſame as employing the whole ſum at

2. What is the present worth of 150 *l.* payable in $\frac{1}{4}$ year, discounting at five *per cent.?* *Ans.* 148 *l.* 2 *s.* 11$\frac{1}{2}$ *d.*
3. What is the present worth of 75 *l.* due 15 months hence, *at* 5 *per cent.?* *Ans.* 70 *l.* 11 *s.* 9 *d.*
4. What is the discount on 85 *l.* 10 *s.* due September 8, this being July 4, reckoning interest at 5 *per cent. per annum?* *Ans.* 15 *s.* 3$\frac{1}{2}$ *d.*
5. What ready money will discharge a debt of 543 *l.* 7 *s.* due 4 months and 18 days hence, at 4$\frac{5}{8}$ *per cent. per annum?* *Ans.* 533 *l.* 18 *s.* 1$\frac{1}{2}$ *d.*
6. Bought a quantity of goods for 150 *l.* ready money, and sold them again for 200 *l.* payable $\frac{3}{4}$ of a year hence; what was the gain in ready money, supposing discount to be made at 5 *per cent.?* *Ans.* 42 *l.* 15 *s.* 5 *d.*
7. What is the present worth of 120 *l.* payable as follows; viz. 50 *l.* at 3 months, 50 *l.* at 5 months, and the rest at 8 months, discounting at 6 *per cent.?* *Ans.* 117 *l.* 5 *s.* 5 *d.*

COMPOUND INTEREST.

COMPOUND INTEREST is that which arises from the principal and interest taken together, as it becomes due, at the end of each stated time of payment.

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at interest till the time the debt becomes due arrives: for if the discount allowed for present payment be put out to interest for that time, its amount will be the same as the interest of the whole debt for the same time: thus, the discount of 105 *l.* due one year hence, reckoning interest at 5 *per cent.* will be 5 *l.* and 5 *l.* put out to interest at 5 *per cent.* for one year, will amount to 5 *l.* 5 *s.* which is exactly equal to the interest of 105 *l.* for one year at 5 *per cent.*

The truth of the rule for working is evident from the nature of simple interest: for since the debt may be considered as the amount of some principal (called here the present worth) at a certain rate *per cent.* and for the given time, that amount must be in the same proportion, either to its principal or interest, as the amount of any other sum, at the same rate, and for the same time, is to its principal or interest.

The method used amongst Bankers, &c. in discounting bills, is to find the interest of the sum drawn for, from the time the bill is discounted, to the time when it becomes due, (including the days of grace) which interest they reckon as the discount, and by that means make it more than it really is.

But when goods are bought or sold, and discount is to be made for present payment, at any rate *per cent.* without regard to time, the interest of the sum, as calculated for a year, is the discount.

RULE

RULE *.

1. Find the amount of the given principal, for the time of the first payment, by simple interest.

2. Consider this amount as the principal for the second payment, whose amount calculate as before, and so on through all the payments to the last, still accounting the last amount as the principal for the next payment.

EXAMPLES.

1. What is the amount of 320 *l.* 10 *s.* for four years, at 5 *per cent. per annum*, compound interest?

	l.	*s.*	*d.*	
$\frac{1}{20}$)	320 *l.*	10 *s.*		*1st. year's principal.*
	16	—	6	*1st. year's interest.*
$\frac{1}{20}$)	336	10	6	*2d. year's principal.*
	16	16	$6\frac{1}{4}$	*2d. year's interest.*
$\frac{1}{20}$)	353	7	$\frac{1}{4}$	*3d. year's principal.*
	17	13	4	*3d. year's interest.*
$\frac{1}{20}$	371	—	$4\frac{1}{4}$	*4th. year's principal.*
	18	11	-	*4th. year's interest.*
	389	11	$4\frac{1}{4}$	*whole amount, or the answer required.*

2. What is the compound interest of 760 *l.* 10 *s.* forborn 4 years, at 4 *per cent.* *Ans.* 129 *l.* 3 *s.* $6\frac{1}{4}$ *d.*

3. What is the amount of 15 *l.* 10 *s.* for 9 years, at $3\frac{1}{2}$ *per cent. per annum*, compound interest? *Ans.* 21 *l.* 2 *s.* $4\frac{1}{4}$ *d.*

4. What is the compound interest of 410 *l.* forborn for $2\frac{1}{2}$ years, at $4\frac{1}{2}$ *per cent. per annum*; the interest payable half yearly? *Ans.* 48 *l.* 4 *s.* $11\frac{1}{4}$ *d.*

Find the several amounts of 50 *l.* payable yearly, half yearly and quarterly, being forborn 5 years, at 5 *per cent. per annum*, compound interest?

Ans. 63 *l.* 16 *s.* $2\frac{3}{4}$ *d.* 64 *l.* 0 *s.* 0 *d.* and 94 *l.* 1 *s.* $9\frac{1}{2}$ *d.*

EQUATION OF PAYMENTS.

EQUATION OF PAYMENTS is the finding a time, to pay at once, several debts due at different times, so that no loss shall be sustained by either party.

* The reason of this rule is evident from the definition, and the principles of simple interest.

RULE*.

Multiply each payment by the time at which it is due; then divide the sum of the products by the sum of the payments, and the quotient will be the time required.

EXAMPLES.

A owes B 190 *l.* to be paid as follows, viz. 50 *l.* in 6 months, 60 *l.* in 7 months, and 80 *l.* in 10 months; what is the equated time to pay the whole?

$$50 \times 6 = 300$$
$$60 \times 7 = 420$$
$$80 \times 10 = 800$$

$$50 + 60 + 80 = 190)\ 1520\ (8$$
$$1520$$

Answer 8 *months.*

2. A owes B 52 *l.* 7 *s.* 6 *d.* to be paid in $4\frac{1}{2}$ months, 80 *l.* 10 *s.* to be paid in $3\frac{1}{2}$ months, and 76 *l.* 2 *s.* 6 *d.* to be paid in 5 months; what is the equated time to pay the whole?

Ans. 4 *mo.* 8 *da.*

3. A owes B 240 *l.* to be paid in 6 months, but in 1 month and a half pays him 60 *l.* and in $4\frac{1}{2}$ months after that 80 *l.* more: how much longer than 6 months should B in equity defer the rest?

Ans. $3\frac{13}{14}$ *months.*

4. A

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* This rule is founded upon a supposition, that the sum of the interests of the several debts which are payable before the equated time, from their terms to that time, ought to be equal to the sum of the interests of the debts payable after the equated time, from that time to their terms. Among others that defend this principle, Mr. Cocker endeavours to prove it to be right by this argument: that what is gained by keeping some of the debts after they are due, is lost by paying others before they are due: but this cannot be the case; for though by keeping a debt unpaid after it is due, there is gained the interest of it for that time, yet by paying a debt before it is due, the payer does not lose the interest for that time, but the discount only, which is less than the interest, and therefore the rule is not true.

Although this rule is not accurately true, yet in most questions that occur in business, the error is so trifling, that it will always be made use of as the most eligible method.

4. A debt is to be paid as follows: viz. $\frac{1}{4}$ at 2 months, $\frac{1}{8}$ at 3 months, $\frac{1}{8}$ at 4 months, $\frac{1}{8}$ at 5 months, and the rest at 7 months: what is the equated time to pay the whole?

Anſ. 4 months and 18 *days.*

5. A owes B 100*l.* to be paid in 9 months, and 500*l.* to be paid in a year and a half: when is the equated time to pay the whole?

6. A debt of 1000*l.* is to be paid as follows: viz. $\frac{1}{2}$ at 8 months, $\frac{1}{3}$ at 12 months, and the reſt in $1\frac{1}{2}$ years: what is the equated time to pay the whole?

BARTER.

BARTER is the exchanging of one commodity for another; and directs traders ſo to proportion their goods, that neither party may ſuſtain loſs.

RULE*.

Find the value of that commodity whoſe quantity is given; then find what quantity of the other, at the rate propoſed, you may have for the ſame money, and it gives the anſwer required.

That the rule is univerſally agreeable to the ſuppoſition, may be thus demonſtrated.

Let $\begin{cases} d = \text{firſt debt payable, and the diſtance of its term of payment } t. \\ D = \text{laſt debt payable, and the diſtance of its term } T. \\ x = \text{diſtance of the equated time.} \\ r = \text{rate of intereſt of 1 } l. \text{ for one year.} \end{cases}$

Then, ſince x lies between T and t. $\begin{cases} \text{The diſtance of the time } t \text{ and } x \text{ is } = x - t. \\ \text{The diſtance of the time T and } x \text{ is } = T - x. \end{cases}$

Now the intereſt of d for the time $x-t$ is $(x-t) \times dr$.; and the intereſt of D for the time $T-x$ is $(T-x) \times Dr$; therefore $(x-t) \times dr = (T-x) \times Dr$ by the ſuppoſition; and from this equation x is found $= \dfrac{DT + dt}{D + d}$, which is the rule. And the ſame might be ſhewn f any number of payments.

The true rule will be given in equation of payments by decimals.

* This rule is, evidently, only an application of the rule of three direct.

EXAM-

EXAMPLES.

1. How many dozen of candles, at 5 *s. per doz.* must be given in barter for 3 *cwt.* of tallow, at 1 *l.* 17 *s.* 4 *d. per cwt.*?

cwt.		*l.*	*s.*	*d.*		*cwt.*
1	:	1	17	4	::	3
4				12		4
4		4)22	8	0		12
		5	12	0		
		20				
		5)112				
		22 *doz.*—$5\frac{4}{5}$ *lb.*				

Ans. 22 *doz.* $5\frac{4}{5}$ *lb.*

2. How much sugar, at 8 *d. per lb.* must be given in barter for 20 *cwt.* of tobacco, at 3 *l. per cwt.*? *Ans.* 16 *cwt.* 0 *qrs.* 8 *lb.*

3. How much tea at 9 *s. per lb.* can I have in barter for 4 *cwt.* 2 *qrs.* of chocolate, at 4 *s. per lb.*? *Ans.* 2 *cwt.*

4. How many reams of paper, at 2 *s.* $9\frac{1}{2}$ *d. per* ream, must be given in barter for 37 pieces of Irish cloth, at 1 *l.* 12 *s.* 4 *d. per piece*? *Ans.* $428\frac{36}{67}$.

5. A merchant hath 1000 yards of canvas at $9\frac{1}{2}$ *d. per yard.* which he barters for serge at $10\frac{1}{4}$ *d. per yard*; how many yards must he receive? *Ans.* $926\frac{34}{41}$.

6. A delivered 3 *hhds.* of brandy, at 6 *s.* 8 *d. per gall.* to B, for 126 yards of cloth; what was the cloth *per yard*? *Ans.* 10 *s.*

7. A and B barter; A hath 41 *cwt.* of hops, at 30 *s. per cwt.* for which B gives him 20 *l.* in money, and the rest in prunes at 5 *d. per lb.* what quantity of prunes must A receive? *Ans.* 17 *cwt.* 3 *qrs.* 4 *lb.*

8. A has a quantity of pepper, wt. neat 1600 *lb.* at 17 *d. per lb.* which he barters with B for two sorts of goods, the one at 5 *d.* the other at 8 *d. per lb.* and to have $\frac{1}{3}$ in money, and of each sort of goods an equal quantity: how many *lb.* of each must he receive, and how much in money? *Ans.* $1394\frac{38}{39}$ *lb. and* 37 *l.* 15 *s.* $6\frac{2}{3}$ *d.*

LOSS AND GAIN.

Loss and Gain is a rule that diſcovers what is got or loſt in the buying or ſelling of goods; and inſtructs merchants and traders to raiſe or fall the price of their commodities, ſo as to gain or loſe ſo much *per cent.* &c.

Queſtions in this rule are performed by the rule of three direct.

Examples.

1. How muſt I ſell tea *per lb.* that coſt me 13 *s.* 5 *d.* to gain after the rate of 25 *per cent.*?

£.		£.		s.	d.
100	:	125	::	13	5
		161		12	
		125		161	
		750			
		125			

1,00)201,25

12)201—25

16 *s.* 9 *d.*—$\frac{25}{100}$ *the anſwer.*

Or thus,

4)13 *s.* 5 *d.*
3 $4\frac{1}{4}$

16 *s.* $9\frac{1}{4}$ *d.* *the ſame as before.*

2. At $1\frac{1}{2}$ *d.* in the ſhilling profit, how much *per cent.*? *Anſ.* 12 *l.* 10 *s.*

3. At 3 *s.* 6 *d.* in the pound profit, how much *per cent.*? *Anſ.* 17 *l.* 10 *s.*

4. If a *lb.* of tobacco coſt 16 *d.* and is ſold for 20 *d.* what is the gain *per cent.*? *Anſ.* 25 *l.*

5. Bought goods at $4\frac{1}{2}$ *d. per lb.* and ſold them at the rate of 2 *l.* 7 *s.* 4 *d. per cwt.* what was the gain *per cent.*? *Anſ.* 12 *l.* 13 *s.* 11 *d.*

6. Bought cloth at 7 *s.* 6 *d. per yard,* which not proving ſo good as I expected, I am reſolved to loſe $17\frac{1}{2}$ *per cent.* by it: how muſt I ſell it *per yard*? *Anſ.* 6 *s.* $2\frac{1}{4}$ *d.*

7. Bought goods at 2 guineas *per cwt.* and ſold them again retail at $5\frac{1}{4}$ *d. per lb.* what was the gain *per cent.*? *Anſ.* 16 *l.* 13 *s.* 4 *d.*

8. If

8. If I buy $17\frac{1}{2}$ *cwt.* of ſugar for 35 guineas, and retail it at $7\frac{1}{2}$ *d. per lb.* what ſhall I gain *per cent.*?

Anſ. 66 *l.* 13 *s.* 4 *d.*

9. If I buy tobacco at 10 guineas *per cwt.* at what rate muſt I retail it *per lb.* to gain twelve *per cent.*?

Anſ. 2 *s.* 1 *d.* $\frac{3}{5}$.

10. If, when I ſell cloth at 7 *s. per yard*, I gain 10 *per cent.* what will be the gain *per cent.* when it is ſold for 8 *s.* 6 *d. per yard*? *Anſ.* 33 *l.* 11 *s.* $5\frac{1}{11}$ *d.*

11. If I buy 28 pieces of ſtuffs at 4 *l. per piece*, and ſell 10 of the pieces at 6 *l.* and 8 at 5 *l. per piece*: at what rate *per piece* muſt I ſell the reſt to gain 20 *per cent.* by the whole? *Anſ.* 2 *l.* 6 *s.* $10\frac{1}{4}$ *d.*

12. Bought 40 gallons of brandy at 3 *s. per gall.* but by accident 6 gallons of it were loſt; at what rate muſt I ſell the remainder *per* gallon, to gain upon the whole prime coſt, at the rate of 10 *per cent.*? *Anſ.* 3 *s.* $10\frac{1}{2}$ *d.*

13. Bought hoſe in London at 4 *s.* 3 *d. per pair*, and ſold them afterwards in Dublin at 6 *s.* the pair; now taking the charge at an average to be 2 *d.* the pair, and conſidering that I muſt loſe 12 *per cent.* by remitting my money home again; what do I gain *per cent.* by this article of trade?

Anſ. 19 *l.* 10 *s.* 11 *d.*

14. Sold a repeating watch for 50 guineas, and by ſo doing loſt 17 *per cent.* whereas I ought in dealing to have cleared 20 *per cent.* how much was it ſold for under the juſt value?

Anſ. 23 *l.* 8 *s.* $0\frac{3}{4}$ *d.*

FELLOWSHIP.

Fellowship is a general rule, by which merchants, &c. trading in company with a joint ſtock, are enabled to aſcertain each perſon's particular ſhare of the gain or loſs, in proportion to his ſhare in the joint ſtock.

By this rule a bankrupt's eſtate may be divided amongſt his creditors, as alſo legacies adjuſted, when there is a deficiency of aſſets or effects.

Single Fellowship.

Single Fellowship is when different ſtocks are employed for any certain equal time.

RULE*.

As the whole ſtock is to the whole gain or loſs,
So is each man's particular ſtock, to his particular ſhare of the gain or loſs.

METHOD OF PROOF.

Add all the ſhares together, and the ſum will be equal to the gain or loſs, when the work is right.

EXAMPLES.

1. Two perſons trade together, A put into ſtock 130*l.* and B 220*l.* and they gained 500*l.* what is each perſon's ſhare thereof?

$$130l. + 220 = 350l.$$

$$\begin{array}{cccccc} \underline{350l.} & : & \underline{500l.} & :: & 130l. \\ 7 & : & 10 & :: & 130 \\ & & \underline{130} & & \\ & & 7)\underline{1300} & & \\ & & 185l.\ 14s.\ 3\tfrac{1}{4}d.\ \tfrac{5}{7} & & \\ 7 & : & 10 & :: & 220 \\ & & \underline{220} & & \\ & & 7)\underline{2200} & & \\ & & 314l.\ 5s.\ 8\tfrac{1}{2}d.\ \tfrac{2}{7} & & \end{array}$$

$$\begin{array}{lllll} 185l. & 14s. & 3\tfrac{1}{4}d.\ \tfrac{5}{7} & = & \textit{A's share.} \\ 314l. & 5s. & 8\tfrac{1}{2}d.\ \tfrac{2}{7} & = & \textit{B's share.} \\ \hline 500l. & 0s. & 0d. & & \textit{the Proof.} \end{array}$$

* That the gain or loſs in this rule, is in proportion to their ſtocks, is evident: for, as the times the ſtocks are in trade are equal, if I put in $\frac{1}{2}$ of the whole ſtock, I ought to have $\frac{1}{2}$ of the whole gain; if my part of the whole ſtock be $\frac{1}{3}$, my ſhare of the whole gain or loſs ought to be $\frac{1}{3}$ alſo. And, generally, if I put in $\frac{1}{n}$ of the ſtock, I ought to have $\frac{1}{n}$ part of the whole gain or loſs; that is, the ſame ratio that the whole ſtock has to the whole gain or loſs, muſt each perſon's particular ſtock have to his reſpective gain or loſs.

2. A

2. A and B have gained by trading 182 *l.* A put into stock 300 *l.* and B 400 *l.* what is each person's share of the profit? *Ans.* A 78 *l.* *and* B 104 *l.*

3. Divide 120 *l.* between three persons, so that their shares shall be to each other as 1, 2 and 3 respectively.
Ans. 20 *l.* 40 *l.* *and* 60 *l.*

4. Three persons make a joint stock; A put in 184 *l.* 10 *s.* B 96 *l.* 15 *s.* and C 76 *l.* 5 *s.* they trade and gain 220 *l.* 12 *s.* what is each person's share of the gain?
Ans. A 113 *l.* 16 *s.* $\frac{688}{715}$, B 59 *l.* 14 *s.* $\frac{12}{715}$, C 47 *l.* 1 *s.* $\frac{15}{715}$.

5. Four persons in partnership, A, B, C and D, put into stock 180 *l.* 240 *l.* 350 *l.* and 430 *l.* respectively, for 5 years certain, and at the end of that time they find they have gained 3600 *l.* what is each person's share of the gain?
Ans. A 540 *l.* B 720 *l.* C 1050 *l.* *and* D 1290 *l.*

6. Three merchants, A, B and C, freight a ship with 340 tuns of wine; A loaded 110 tun, B 97, and C the rest. In a storm the seamen were obliged to throw 85 tuns overboard; how much must each sustain of the loss?
Ans. A $27\frac{1}{2}$, B $24\frac{1}{4}$, *and* C $33\frac{1}{4}$.

7. A ship worth 860 *l.* being entirely lost, of which $\frac{1}{8}$ belonged to A, $\frac{1}{4}$ to B, and the rest to C; what loss will each sustain, supposing 500 *l.* of her to be insured?
Ans. A 45 *l.* B 90 *l.* *and* C 225 *l.*

8. A bankrupt is indebted to A 275 *l.* 14 *s.* to B 304 *l.* 7 *s.* to C 152 *l.* and to D 104 *l.* 6 *s.* His estate is worth only 675 *l.* 15 *s.* how must it be divided?
Ans. A 222 *l.* 15 *s.* 2 *d.* B 245 *l.* 18 *s.* $1\frac{1}{2}$ *d.* C 122 *l.* 16 *s.* $2\frac{3}{4}$ *d.* *and* D 84 *l.* 5 *s.* 5 *d.*

9. A and B venturing equal sums of money, clear by joint trade 154 *l.* By agreement A was to have 8 *per cent.* because he spent his time in the execution of the project, and B was to have only 5 *per cent.*; what was A allowed for his trouble? *Ans.* 35 *l.* 10 *s.* $9\frac{3}{13}$.

10. A person ordered 1000 *l.* to be divided among his three sons, so that A might have $\frac{1}{3}$ part, B $\frac{1}{4}$ and C $\frac{1}{5}$: what is the just share of each?

11. Three merchants, in partnership, as A, B and C, put into stock 2000 *l.* 3500 *l.* and 4550 *l.* respectively, for 3 years certain, and, at the end of that time, find they have cleared 10,000 *l.* what is each person's share of the gain?

DOUBLE FELLOWSHIP.

DOUBLE FELLOWSHIP is when different or equal ſtocks are employed for different times.

RULE *.

Multiply each man's ſtock into the time of its continuance; then ſay,

As the total ſum of all the products is to the whole gain or loſs;

So is each man's particular product, to his particular ſhare of the gain or loſs.

EXAMPLES.

1. A and B hold a piece of ground in common, for which they are to pay 36 *l.* A put in 23 oxen for 27 days, and B 21 oxen for 39 days; what ought each man to pay of the rent?

$$23 \times 27 = 621$$
$$21 \times 39 = 819$$
$$\overline{1440}$$

1440 : 36 :: 621

240 : 6

40 : 1

621

4,0)62,1

15 *l.* 10 *s.* 6 *d.*

40 : 1 :: 819

819

4,0)81,9

20 *l.* 9 *s.* 6 *d.*

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* Mr. Malcolm, Mr. Ward, and ſeveral other authors, have given an analytical inveſtigation of this rule; but the moſt general and elegant method I have met with is that by Mr. Hutton in p. 88 of his arithmetic, viz.

When the times are equal, the ſhares of the gain or loſs are evidently as the ſtocks, as in Single Fellowſhip; and when the ſtocks are equal, the ſhares are as the times; wherefore when neither are equal, the ſhares muſt be as their products.

15 *l.*	10 *s.*	6 *d.*	=	*A's share.*
20 *l.*	9 *s.*	6 *d.*	=	*B's share.*
36 *l.*	0 *s.*	0 *d.*		*the Proof.*

2. A, B and C hold a pasture in common, for which they pay 30 *l.* *per annum.* A put into it 7 oxen for 3 months, B 9 oxen for 5 months, and C 4 for 12 months: what must each pay of the rent? *Ans.* A 5 *l.* 10 *s.* $6\frac{1}{4}$ *d.* $\frac{30}{114}$. B 11 *l.* 16 *s.* 10 *d.* $\frac{48}{114}$, *and* C 12 *l.* 12 *s.* $7\frac{1}{2}$ *d.* $\frac{36}{114}$.

3. Three graziers hired a piece of land for 60 *l.* 10 *s.* A put in 5 sheep for $4\frac{1}{2}$ months, B put in 8 for 5 months, and C put in 9 for $6\frac{1}{2}$ months: how much must each pay of the rent? *Ans.* A 11 *l.* 5 *s.* B 20 *l.* *and* C 29 *l.* 5 *s.*

4. Two merchants enter into partnership for 18 months; A put into stock at first 200 *l.* and at 8 months end he put in 100 *l.* more; B put in at first 550 *l.* and at 4 months end took out 140 *l.* Now at the expiration of the time they find they have gained 526 *l.*: what is each man's just share? *Ans.* A 192 *l.* 19 *s.* 0 *d.* $\frac{672}{1234}$. B 333 *l.* 0 *s.* $11\frac{3}{4}$ *d.* $\frac{582}{1234}$.

5. A with a capital of 1000 *l.* began trade January 1st, 1776, and, meeting with success in business, took in B as a partner, with a capital of 1500 *l.* on the 1st of March following. Three months after that they admit C as a third partner, who brought into stock 2800 *l.* and after trading together till the first of the next year, they find there has been gained, since A's commencing of business, 1776 *l.* 10 *s.*: how must this be divided amongst the partners? *Ans.* A 457 *l.* 9 *s.* $4\frac{1}{4}$ *d.* B 571 *l.* 16 *s.* $8\frac{1}{4}$ *d.* C. 747 *l.* 3 *s.* $11\frac{1}{4}$ *d.*

ALLIGATION.

ALLIGATION teaches how to mix several simples of different qualities, so that the composition may be of a middle quality; and is commonly distinguished into two principal cases, called Alligation medial, and Alligation alternate.

ALLIGATION MEDIAL.

ALLIGATION MEDIAL is the method of finding the rate of the compound, from having the rates and quantities of the several simples given.

RULE.

RULE*.

Multiply each quantity by its rate; then divide the ſum of the products by the ſum of the quantities, or the whole compoſition, and the quotient will be the rate of the compound required.

EXAMPLES.

1. Suppoſe 15 buſhels of wheat at 5 *s.* *per* buſhel, and 12 buſhels of rye at 3 *s.* 6 *d.* *per* buſhel were mixed together: how muſt the compound be ſold *per* buſhel without loſs or gain?

60	42	15
15	12	12
300	504	27
60	900	
900	27)1404(52 *d.* = 4 *s.* 4 *d.* *the anſwer.*	
	135	
	54	
	54	

2. A compoſition being made of 5 *lb.* of tea at 7 *s.* *per lb.* 9 *lb.* at 8 *s.* 6 *d.* *per lb.* and $14\frac{1}{2}$ *lb.* at 5 *s.* 10 *d.* *per lb.* what is a *lb.* of it worth? *Anſ.* 6 *s.* $10\frac{1}{2}$ *d.*

3. Mixed 4 gallons of wine at 4 *s.* 10 *d.* *per gall.* with 7 gallons at 5 *s.* 3 *d.* *per gall.* and $9\frac{1}{4}$ gallons at 5 *s.* 8 *d.* *per gall.* what is a gallon of this compoſition worth? *Anſ.* 5 *s.* $4\frac{1}{4}$ *d.*

4. A mealman would mix 3 buſhels of flour at 3 *s.* 5 *d.* *per* buſhel, 4 buſhels at 5 *s.* 6 *d.* *per* buſhel, and 5 buſhels at 4 *s.* 8 *d.* *per* buſhel: what is the worth of a buſhel of this mixture? *Anſ.* 4 *s.* $7\frac{1}{2}$ *d.*

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* The truth of this rule is too evident to need a demonſtration.

Note, If an ounce or any other quantity of pure gold be reduced into 24 equal parts, theſe parts are called caracts; but gold is often mixed with ſome baſer metal, which is called the alloy, and the mixture is ſaid to be of ſo many caracts fine, according to the proportion of pure gold contained in it: thus, if 22 caracts of pure gold, and 2 of alloy are mixed together, it is ſaid to be 22 caracts fine.

If any one of the ſimples be of little or no value with reſpect to the reſt, its rate is ſuppoſed to be nothing, as water mixed with wine, and alloy with gold and ſilver.

5. A farmer mixes 20 bushels of wheat at 5 *s.* *per* bushel, and 36 bushels of rye at 3 *s.* *per* bushel, and 40 bushels of barley at 2 *s.* *per* bushel: what is the worth of a bushel of this mixture? *Ans.* 3 *s.*

6. A goldsmith melts 8 *lb.* $5\frac{1}{2}$ *oz.* of gold bullion of 14 caracts fine, with 12 *lb.* $8\frac{1}{2}$ *oz.* of 18 caracts fine: how many caracts fine is this mixture? *Ans.* $16\frac{204}{508}$ *caracts.*

7. A refiner melts 10 *lb.* of gold of 20 caracts fine with 16 *lb.* of 18 caracts fine; how much alloy must he put to it to make it 22 caracts fine?

Ans. It is not fine enough by $3\frac{6}{26}$ *caracts, so that no alloy must be put to it, but more gold.*

ALLIGATION ALTERNATE.

ALLIGATION ALTERNATE is the method of finding what quantity of any number of simples, whose rates are given, will compose a mixture of a given rate; so that it is the reverse of alligation medial, and may be proved by it.

RULE I*.

1. Write the rates of the simples in a column under each other.

2. Connect, or link with a continued line, the rate of each simple, which is less than that of the compound, with one, or any number, of those that are greater than the compound; and each greater rate with one or any number of the less.

3. Write the difference between the mixture rate, and that of each of the simples, opposite the rates with which they are linked.

4. Then if only one difference stand against any rate, it will be the quantity belonging to that rate; but if there be several, their sum will be the quantity.

EXAM-

* *Demon.* By connecting the less rate to the greater, and placing the differences between them and the mean rate alternately, the quantities resulting are such, that there is precisely as much gained by one quantity as is lost by the other, and therefore the gain and loss upon the whole is equal, and is exactly the proposed rate: and the same will be true of any other two simples managed according to the rule.

In like manner, let the number of simples be what they will, and with how many soever every one is linked, since it is always a less with a greater than the mean price, there will be an equal balance of loss and gain

RULE*.

Multiply each quantity by its rate; then divide the sum of the products by the sum of the quantities, or the whole composition, and the quotient will be the rate of the compound required.

EXAMPLES.

1. Suppose 15 bushels of wheat at 5 *s. per* bushel, and 12 bushels of rye at 3 *s.* 6 *d. per* bushel were mixed together: how must the compound be sold *per* bushel without loss or gain?

60	42	15
15	12	12
300	504	27
60	900	
900	27)1404(52 *d.* = 4 *s.* 4 *d. the answer.*	
	135	
	54	
	54	

2. A composition being made of 5 *lb.* of tea at 7 *s. per lb.* 9 *lb.* at 8 *s.* 6 *d. per lb.* and 14½ *lb.* at 5 *s.* 10 *d. per lb.* what is a *lb.* of it worth? *Ans.* 6 *s.* 10½ *d.*
3. Mixed 4 gallons of wine at 4 *s.* 10 *d. per gall.* with 7 gallons at 5 *s.* 3 *d. per gall.* and 9¾ gallons at 5 *s.* 8 *d. per gall.* what is a gallon of this composition worth? *Ans.* 5 *s.* 4¼ *d.*
4. A mealman would mix 3 bushels of flour at 3 *s.* 5 *d. per* bushel, 4 bushels at 5 *s.* 6 *d. per* bushel, and 5 bushels at 4 *s.* 8 *d. per* bushel: what is the worth of a bushel of this mixture? *Ans.* 4 *s.* 7½ *d.*

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* The truth of this rule is too evident to need a demonstration.

Note, If an ounce or any other quantity of pure gold be reduced into 24 equal parts, these parts are called caracts; but gold is often mixed with some baser metal, which is called the alloy, and the mixture is said to be of so many caracts fine, according to the proportion of pure gold contained in it: thus, if 22 caracts of pure gold, and 2 of alloy are mixed together, it is said to be 22 caracts fine.

If any one of the simples be of little or no value with respect to the rest, its rate is supposed to be nothing, as water mixed with wine, and alloy with gold and silver.

5. A

5. A farmer mixes 20 bushels of wheat at 5 *s. per* bushel, and 36 bushels of rye at 3 *s. per* bushel, and 40 bushels of barley at 2 *s. per* bushel: what is the worth of a bushel of this mixture? *Ans.* 3 *s.*

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3. Write the difference between the mixture rate, and that of each of the simples, opposite the rates with which they are linked.

4. Then if only one difference stand against any rate, it will be the quantity belonging to that rate; but if there be several, their sum will be the quantity.

EXAM-

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* *Demon.* By connecting the less rate to the greater, and placing the differences between them and the mean rate alternately, the quantities resulting are such, that there is precisely as much gained by one quantity as is lost by the other, and therefore the gain and loss upon the whole is equal, and is exactly the proposed rate: and the same will be true of any other two simples managed according to the rule.

In like manner, let the number of simples be what they will, and with how many soever every one is linked, since it is always a less with a greater than the mean price, there will be an equal balance of loss and gain

EXAMPLES.

1. A merchant would mix wines at 17*s.* 18*s.* and 22*s. per* gallon, so as that the mixture may be worth 20*s.* the gallon: what quantity of each must be taken?

$$20\left\{\begin{array}{l}17\\18\\22\end{array}\right\}\begin{array}{l}2 \textit{ at } 17s.\\ 2 \textit{ at } 18s.\\ 3+2=5 \text{ at } 22s.\end{array}$$

Ans. 2 gallons at 17*s.* 2 gallons at 18*s.* and 5 at 22*s.*

2. How much wine at 6*s. per* gallon, and at 4*s. per* gallon, must be mixed together, that the composition may be worth 5*s. per* gallon? *Ans. 1 qt. or 1 gall.* &c.

3. How much corn at 2*s.* 6*d.* 3*s.* 8*d.* 4*s.* and 4*s.* 8*d. per* bushel, must be mixed together, that the compound may be worth 3*s.* 10*d. per* bushel? *Ans.* 12 at 2*s.* 6*d.* 12 at 3*s.* 8*d.* 18 at 4*s.* and 18 at 4*s.* 8*d.*

4. A goldsmith has gold of 17, 18, 22, and 24 carats fine: how much must he take of each to make it 21 carats fine? *Ans.* 3 *of* 17, 1 *of* 18, 3 *of* 22, *and* 4 *of* 24.

5. It is required to mix brandy at 8*s.* wine at 7*s.* cyder at 1*s.* and water at 0 *per* gallon together, so that the mixture may be worth 5*s. per* gallon? *Ans.* 9 *gals. of brandy,* 9 *of wine,* 5 *of cyder, and* 5 *of water.*

6. How much sugar at 4*d.* at 6*d.* and at 11*d. per lb.* must be mixed together, so that the composition formed by them may be worth 7*d. per lb.*?

Ans. 1 *lb. or* 1 *stone, or* 1 *cwt. or any other equal quantity of each sort.*

---

gain between every two, and consequently an equal balance on the whole. *Q. E. D.*

It is obvious, from the rule, that questions of this sort admit of a great variety of answers; for, having found one answer, we may find as many more as we please, by only multiplying or dividing each of the quantities found by 2, 3, or 4, &c. the reason of which is evident; for, if two quantities, of two simples, make a balance of loss and gain, with respect to the mean price, so must also the double or treble, the $\frac{1}{2}$ or $\frac{1}{3}$ part, or any other ratio of these quantities, and so on *ad infinitum.*

These kind of questions are called by algebraists *indeterminate* or *unlimited* problems, and, by an analytical process, theorems may be raised that will give all the *possible* answers.

RULE 2.

When the whole composition is limited to a certain quantity.

Find an answe· as before by linking; then say, as the sum of the quantitie., or differences thus determined, is to the given quantity. so is each ingredient, found by linking, to the required quantity of each.

EXAMPLES.

1. How much gold of 15, 17, 18 and 22 caracts fine must be mixed together to form a composition of 40 *oz.* of 20 caracts fine?

$$20\begin{cases}15 & \text{- - - - - -} \quad 2\\ 17 & \text{- - - - - -} \quad 2\\ 18 & \text{- - - - - -} \quad 2\\ 22 & 5+3+2=10\end{cases}$$

$$\overline{16}$$

$$16 : 40 :: \begin{cases} 2 : \dfrac{40\times 2}{16} = \dfrac{80}{16} = 5 \\ 10 : \dfrac{40\times 10}{16} = \dfrac{400}{16} = 25 \end{cases}$$

Ans. 5 *oz.* of 15, 17 and 18 caracts fine, and 25 *oz.* of 22 caracts fine.

2. A grocer has currants at 4*d.* 6*d.* 9*d.* and 11*d.* *per lb.* and he would make a mixture of 240 *lb.* so that it might be afforded at 8*d.* *per lb.* how much of each sort must he take?

Ans. 72 *lb.* *at* 4*d.* 24 *at* 6*d.* 48 *at* 9*d.* *and* 96 *at* 11*d.*

RULE

---

* A great number of questions might be here given relating to the specific gravities of metals, &c. but as they are best performed by fractions, I shall only give one of the most curious, and work out the example at large.

Heiro, king of Syracuse, gave orders for a crown to be made him entirely of pure gold; but suspecting the workman had debased it by mixing it with silver or copper, he recommended the discovery of the fraud to the famous Archimedes; and desired to know the exact quantity of alloy in the crown.

Archimedes, in order to detect the imposition, procured two other masses, the one of pure gold, the other of silver or copper, and each of

 the

RULE 3 *.

When one of the ingredients is limited to a certain quantity.

Take the difference between each price, and the mean rate as before; then,

As the difference of that ſimple, whoſe quantity is given, is to the reſt of the differences ſeverally, ſo is the quantity given to the ſeveral quantities required.

EXAMPLES.

1. How much wine at 5 *s.* at 5 *s.* 6 *d.* and 6 *s.* the gallon muſt be mixed with 3 gallons at 4 *s. per gallon,* ſo that the mixture may be worth 5 *s.* 4 *d. per gallon?*

64	48	8	+	2	=	10
	60	8	+	2	=	10
	66	16	+	4	=	20
	72	16	+	4	=	20

10 : 10 :: 3 : 3

10 : 20 :: 3 : 6

10 : 20 :: 3 : 6

Anſ. 3 *gallons at* 5 *s.* 6 *at* 5 *s.* 6 *d. and* 6 *at* 6 *s.*

---

the ſame weight with the former; and by putting each ſeparately into a veſſel full of water, the quantity of water expelled by them determined their ſpecific gravities: from which and their given weights, the exact quantities of gold and alloy in the crown may be determined.

Suppoſe the weight of each crown to be 10 *lb.* and that the water expelled by the copper or ſilver was .92 *lb.* by the gold .52 *lb.* and by the compound crown .64 *lb.* what will be the quantities of gold and alloy in the crown?

The rates of the ſimples are 92 and 52, and of the compound 64: therefore

64	92	12 of copper
	52	28 of gold

And the ſum of theſe is 12 + 28 = 40, which ſhould have been but 10; whence, by the rule,

40 : 10 :: 12 : 3 *lb.* of copper } the anſwer.
40 : 10 :: 28 : 7 *lb.* of gold }

* In the very ſame manner queſtions may be wrought when ſeveral of the ingredients are limited to certain quantities, by finding firſt for one limit, and then for another.

The two laſt rules can want no demonſtration, as they evidently reſult from the firſt, the reaſon of which has been already explained.

2. A

2. A grocer would mix teas at 12*s.* 10*s.* and 6*s.* *per lb.* with 20 *lb.* at 4*s. per lb.* how much of each sort must he take to make the composition worth 8*s. per lb.*?

Ans. 20 *lb. at* 4*s.* 10 *lb. at* 6*s.* 10 *lb. at* 10*s. and* 20 *lb. at* 12*s.*

3. How much gold of 15, of 17, and of 22 caracts fine, must be mixed with 5 *oz.* of 18 caracts fine, so that the composition may be 20 caracts fine?

Ans. 5 *oz. of* 15 *caracts fine,* 5 *oz. of* 17, *and* 25 *of* 22.

VULGAR FRACTIONS.

FRACTIONS, or broken numbers, are expressions for any assignable part or parts of an unit; and are represented by two numbers, placed one above the other, with a line drawn between them.

The figure above the line is called the *numerator,* and that below the line the *denominator.*

The denominator shews how many parts the integer is divided into, and the numerator shews how many of those parts are designed by the fraction.

Fractions are either proper, improper, single, compound, or mixed.

1. A *proper fraction* is when the numerator is less than the denominator, as $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{7}$, &c.

2. An *improper fraction* is when the numerator exceeds the denominator, as $\frac{8}{3}$, $\frac{110}{21}$, &c.

3. A *single fraction* is a simple expression denoting any number of parts of the integer.

4. A *compound fraction* is the fraction of a fraction, as $\frac{1}{2}$ of $\frac{2}{3}$, $\frac{3}{4}$ of $\frac{5}{6}$, &c.

5. A *mixed number* is that which is composed of a whole number and a fraction, as $8\frac{1}{5}$, $17\frac{6}{13}$, &c.

Note, any whole number may be expressed like a fraction, by writing 1 underneath it.

6. The *common measure* of two or more numbers, is that number which will divide each of them without a remainder. Thus, 3 is the common measure of 12 and 15; and the *greatest* number that will do this is called the *greatest common measure.*

7. A number which can be measured by two or more numbers, is called their *common multiple*; and if it be the *least* number which can be so measured, it is called their

least common multiple; thus 30, 45, 60 and 75, are multiples of 3 and 5; but their least common multiple is 15 *.

PROBLEM I.

To find the greatest common measure of two or more numbers.

RULE †.

1. If there be two numbers only, divide the greater by the less, and this divisor by the remainder, and so on, always dividing the last divisor by the last remainder, till nothing remains, then will the last divisor be the greatest common measure required.

2. When there are more than two numbers, find the greatest common measure of two of them as before; and of that common measure and one of the other numbers; and so on, through all the numbers to the last; then will the greatest common measure last found be the answer.

3. If 1 is found to be the common measure, the given numbers are prime to each other, or what are usually called incommensurable.

EXAM-

* A *prime number* is that which can only be measured by an unit.

That number which is produced by multiplying several numbers together, is called a *composite number*.

A *perfect number* is equal to the sum of all its aliquot parts.

The following perfect numbers are taken from the Petersburg acts, and are all that are known at present.

6
28
496
8128
33550336
8589869056
137438691328
2305843008139952128
2417851639228158837784576
9903520314282971830448816128

There are several other numbers which have received different denominations, but they are principally of use in Algebra, and the higher parts of the mathematics.

† This and the following problem will be found very useful in the doctrine of fractions, and several other parts of Arithmetic.

The truth of the rule may be shewn from the 1st example. For since 54 measures 108, it also measures 108 + 54, or 162.

Again,

EXAMPLES.

1. Required the greatest common measure of 918, 1998, and 522.

```
918)1998(2          So 54 is the greatest common measure of
    1836                 1998 and 918
    ----
     162)918(5      Hence 54)522(9
         810                 486
         ---                 ---
         108)162(1            36)54(1
             108                 36
             ---                 --
              54)108(2           18)36(2
                 108                36
                 ---                --
```

Therefore 18 *is the answer required.*

2. What is the greatest common measure of 612 and 540? *Ans.* 36

3. What is the greatest common measure of 720, 336 and 1736? *Ans.* 8

PROBLEM 2.

To find the least common multiple of two or more numbers.

RULE*.

1. Divide by any number that will divide two or more of the given numbers without a remainder, and set the quotients, together with the undivided numbers, in a line below them.

2. Divide

Again, since 54 measures 108, and 162, it also measures $(5 \times 162) + 108$ or 918. In the same manner it will be found to measure $(2 \times 918) + 162$ or 3698, and so on. Therefore 84 measures both 918 and 1998.

It is also the greatest common measure; for suppose there be a greater, then since the greater measures 918 and 1998, it also measures the remainder 162; and since it measures 162 and 918, it also measures the remainder 108; in the same manner it will be found to measure the remainder 54; that is, the greater measures the less, which is absurd. Therefore 54 is the greatest common measure.

In the very same manner the demonstration may be applied to any other numbers.

* The reason of this rule, may, also, be shewn from the 1st example, thus: it is evident that $3 \times 5 \times 8 \times 10 = 1200$ may be divided by 3, 5, 8 and 10, without a remainder; but 10 is a multiple of 5, therefore

2. Divide the ſecond line as before, and ſo on till there are no two numbers that can be divided; then the continued product of the diviſors and quotients will give the multiple required.

EXAMPLES.

1. What is the leaſt common multiple of 3, 5, 8 and 10?

$$\begin{array}{r|rrrr} 5) & 3, & 5, & 8, & 10 \\ \hline 2) & 3, & 1, & 8, & 2 \\ \hline & 3, & 1, & 4, & 1 \end{array}$$

$5 \times 2 \times 3 \times 4 = 120$ *the anſwer.*

2. What is the leaſt common multiple of 4 and 6? *Anſ.* 12

3. What is the leaſt number that 3, 4, 8 and 12 will meaſure? *Anſ.* 24

4. What is the leaſt number that can be divided by the nine digits, without a remainder? *Anſ.* 2520

REDUCTION OF VULGAR FRACTIONS.

REDUCTION OF VULGAR FRACTIONS is the bringing them out of one form or denomination into another, in order to prepare them for the operations of addition, ſubtraction, &c.

CASE I.

To abbreviate or reduce fractions to their loweſt terms.

RULE*.

Divide the terms of the given fraction by any number that will divide them without a remainder, and theſe quotients again in the ſame manner; and ſo on, till it appears that there is no number greater than 1, which will divide them, and the fraction will be in its loweſt terms.

Or,

fore $3 \times 5 \times 8 \times 2$, or 240, is alſo diviſible by 5, 5, 8 and 10. Alſo 8 is a multiple of 2; therefore $3 \times 5 \times 4 \times 2 = 120$ is alſo diviſible by 3, 5, 8 and 10; and is, evidently, the leaſt number that can be ſo divided.

* That dividing both the terms of the fraction, equally, by any number whatever, will give another fraction equal to the former, is evident. And if thoſe diviſions are performed as often as can be done, or the common diviſor be the greateſt poſſible, the terms of the reſulting fraction muſt be the leaſt poſſible.

Note,

Or,

Divide both the terms of the fraction by their greateſt common meaſure, and the quotients will be the terms of the fraction required.

EXAMPLES.

1. Reduce $\frac{144}{240}$ to its loweſt terms.

$$\overset{(2)}{\frac{144}{240}} = \overset{(2)}{\frac{72}{120}} = \overset{(3)}{\frac{36}{60}} = \overset{(2)}{\frac{12}{20}} = \overset{(2)}{\frac{6}{10}} = \frac{3}{5}, \text{ the anſwer.}$$

Or thus,

```
144)240(1
    144
    ---
     96)144(1
         96
        ---
         48)96(2
            96
            --
```

Therefore 48 *is the greateſt common meaſure, and* $48)\frac{144}{240} = \frac{3}{5}$ *the ſame as before.*

2. Reduce $\frac{48}{272}$ to its leaſt terms. *Anſ.* $\frac{3}{17}$
3. Reduce $\frac{192}{576}$ to its loweſt terms. *Anſ.* $\frac{1}{3}$
4. Reduce $\frac{825}{960}$ to its leaſt terms. *Anſ.* $\frac{55}{64}$
5. Reduce $\frac{252}{364}$ to its loweſt terms. *Anſ.* $\frac{9}{13}$
6. Reduce $\frac{5184}{6012}$ to its leaſt terms. *Anſ.* $\frac{144}{167}$
7. Reduce $\frac{1344}{1536}$ to its loweſt terms. *Anſ.* $\frac{7}{8}$
8. Abbreviate $\frac{689680}{3670016}$ as much as poſſible.

Anſ. $\frac{43105}{229376}$

CASE

Note, 1. Any number ending with an even number, or a cypher, is diviſible by 2.

2. Any number ending with 5, or 0, is diviſible by 5.

3. If the right-hand place of any number be 0, the whole is diviſible by 10.

4. If the two right-hand figures of any number are diviſible by 4, the whole is diviſible by 4.

5. If the three right-hand figures of any number are diviſible by 8, the whole is diviſible by 8.

6. If the ſum of the digits conſtituting any number be diviſible by 3, or 9, the whole is diviſible by 3, or 9.

7. If the right-hand digit be even, and the ſum of all the digits be diviſible by 6, the whole will be diviſible by 6.

8. A

CASE 2.

To reduce a mixed number to its equivalent improper fraction.

RULE *.

Multiply the whole number by the denominator of the fraction, and add the numerator to the product; then that sum written above the denominator will form the fraction required.

EXAMPLES.

1. Reduce $27\frac{2}{9}$ to its equivalent improper fraction.

$$\begin{array}{r} 27 \\ 9 \\ \hline 243 \\ 2 \\ \hline 245 \\ \hline 9 \end{array}$$

Or $\frac{(27 \times 9) + 2}{9} = \frac{245}{9}$ *the answer.*

2. Reduce $183\frac{5}{21}$ to its equivalent improper fraction.

Ans. $\frac{3848}{21}$

3. Reduce

---

8. A number is divisible by 11, when the sum of the 1st, 3d, 5th, &c. digits is equal to the sum of the 2d, 4th and 6th.

9. If a number cannot be divided by some number less than the square root thereof, that number is a prime.

10. All prime numbers, except 2 and 5, have 1, 3, 7 or 9 in the place of units; and all other numbers are composite.

11. When numbers, with the sign of addition or subtraction between them, are to be divided by any number, each of the numbers must be divided. Thus $\frac{4 + 8 + 10}{2} = 2 + 4 + 5 = 11$.

12. But if the numbers have the sign of multiplication between them, only one of them must be divided. Thus $\frac{3 \times 8 \times 10}{2 \times 6} = \frac{3 \times 4 \times 10}{1 \times 6} = \frac{1 \times 4 \times 10}{1 \times 2} = \frac{1 \times 2 \times 10}{1 \times 1} = \frac{20}{1} = 20$.

* All fractions represent a division of the numerator by the denominator, and are taken altogether as proper and adequate expressions for the quotient. Thus the quotient of 2 divided by 3 is $\frac{2}{3}$; from whence the rule

3. Reduce $514\frac{5}{16}$ to an improper fraction.

Anſ. $\frac{8229}{16}$

4. Reduce $100\frac{19}{59}$ to an improper fraction.

Anſ. $\frac{5919}{59}$

5. Reduce $47\frac{3147}{8400}$ to an improper fraction.

Anſ. $\frac{397947}{8400}$

CASE 3.

To reduce an improper fraction to its equivalent whole or mixed number.

RULE*.

Divide the numerator by the denominator, and the quotient will be the whole or mixed number required.

EXAMPLES.

1. Reduce $\frac{981}{16}$ to its equivalent whole or mixed number.

```
16)981(61 5/16
   96
   --
    21
    16
    --
     5
```

Or,

$$\frac{981}{16} = 981 \div 16 = 61\tfrac{5}{16} \text{ the anſwer.}$$

2. Reduce $\frac{56}{8}$ to its equivalent whole or mixed number.

Anſ. 7

3. Reduce $\frac{1245}{22}$ to its equivalent whole or mixed number.

Anſ. $56\frac{13}{22}$

4. Reduce $\frac{3848}{21}$ to its equivalent whole or mixed number.

Anſ. $183\frac{5}{21}$

---

rule is manifeſt; for if any number is multiplied and divided by the ſame number, it is evident the quotient muſt be the ſame as the quantity firſt propoſed.

* This rule is plainly the reverſe of the former, and has its reaſon in the nature of common diviſion.

5. Reduce

5. Reduce $\frac{621613}{514}$ to its equivalent whole or mixed number.

Ans. $1209\frac{187}{514}$

CASE 4.

To reduce a whole number to an equivalent fraction, having a given denominator.

RULE*.

Multiply the whole number by the given denominator, and place the product over the said denominator, and it will form the fraction required.

EXAMPLES.

1. Reduce 7 to a fraction whose denominator shall be 9.

$7 \times 9 = 63$, *and* $\frac{63}{9}$ *the answer.*

And $\frac{63}{9} = 63 \div 9 = 7$ *the proof.*

2. Reduce 13 to a fraction whose denominator shall be 12.

Ans. $\frac{156}{12}$

3. Reduce 100 to a fraction whose denominator shall be 90.

Ans. $\frac{9000}{90}$

CASE 5.

To reduce a compound fraction to an equivalent simple one.

RULE †.

Multiply all the numerators together for a numerator, and all the denominators together for the denominator; and they will form the simple fraction required.

---

* Multiplication and division are here equally used, and consequently the result is the same as the quantity first proposed.

† That a compound fraction may be represented by a simple one is very evident; since a part of a part must be equal to some part of the whole. The truth of the rule for this reduction may be shewn as follows.

Let the compound fraction to be reduced be $\frac{2}{3}$ of $\frac{4}{7}$. Then $\frac{1}{3}$ of $\frac{4}{7} = \frac{4}{7} \div 3 = \frac{4}{21}$, and consequently $\frac{2}{3}$ of $\frac{4}{7} = \frac{4}{21} \times 2 = \frac{8}{21}$, the same as by the rule, and the like will be found to be true in all cases.

If the compound fraction consists of more numbers than 2, the two first may be reduced to one, and that one and the third will be the same as a fraction of two numbers; and so on.

If part of the compound fraction be a whole or mixed number, it must be reduced to a fraction by one of the former cases.

And when it can be done, any two terms of the fraction may be divided by the same number, and the quotients used instead of them.

EXAMPLES.

1. Reduce $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{8}{11}$ to a simple fraction.

$$\frac{2\times3\times8}{3\times4\times11}=\frac{48}{132}=\frac{4}{11}\text{ the answer.}$$

Or,

$$\frac{2\times\cancel{3}\times\overset{2}{\cancel{8}}}{\cancel{3}\times\cancel{4}\times11}=\frac{4}{11}\text{ as before.}$$

2. Reduce $\frac{4}{7}$ of $\frac{8}{9}$ to a simple fraction. *Ans.* $\frac{32}{63}$
3. Reduce $\frac{2}{3}$ of $\frac{3}{5}$ of $\frac{5}{8}$ to a simple fraction. *Ans.* $\frac{1}{4}$
4. Reduce $\frac{3}{4}$ of $\frac{2}{3}$ of $\frac{4}{11}$ to a simple fraction. *Ans.* $\frac{2}{11}$
5. Reduce $\frac{11}{12}$ of $\frac{7}{13}$ of $\frac{8}{19}$ of 10 to a simple fraction. *Ans.* $\frac{1540}{741}$

CASE 6.

To reduce fractions of different denominators to equivalent fractions, having a common denominator.

RULE I *.

Multiply each numerator into all the denominators but its own, for a new numerator, and all the denominators continually for a common denominator.

---

* By placing the numbers multiplied, properly under one another, it will be seen that the numerator and denominator of every fraction are multiplied by the very same number, and consequently their values are not altered. Thus in the first example;

$$\frac{1}{2}\,\Big|\,\frac{\times5\times7}{\times5\times7}\qquad\frac{3}{5}\,\Big|\,\frac{\times2\times7}{\times2\times7}\qquad\frac{4}{7}\,\Big|\,\frac{\times2\times5}{\times2\times5}$$

In the 2d rule, the common denominator is a multiple of all the denominators, and consequently will divide by any of them; it is manifest, therefore, that proper parts may be taken for all the numerators as required.

EXAM-

EXAMPLES.

1. Reduce $\frac{1}{2}$, $\frac{3}{5}$, and $\frac{4}{7}$ to equivalent fractions, having a common denominator.

$1 \times 5 \times 7 = 35$ *the new numerator for* $\frac{1}{2}$,
$3 \times 2 \times 7 = 42$ *ditto fcr* $\frac{3}{5}$,
$4 \times 2 \times 5 = 40$ *ditto for* $\frac{4}{7}$,
$2 \times 5 \times 7 = 70$ *the common denominator.*

Therefore the new equivalent fractions are $\frac{35}{70}$, $\frac{42}{70}$, *and* $\frac{40}{70}$, *the answer.*

2. Reduce $\frac{1}{2}$, $\frac{2}{3}$, $\frac{5}{6}$ and $\frac{7}{8}$ to fractions, having a common denominator. *Ans.* $\frac{144}{288}$, $\frac{192}{288}$, $\frac{240}{288}$, $\frac{252}{288}$

3. Reduce $\frac{1}{3}$, $\frac{3}{4}$, of $\frac{4}{5}$, $5\frac{1}{2}$ and $\frac{2}{19}$ to a common denominator. *Ans.* $\frac{190}{570}$, $\frac{342}{570}$, $\frac{3135}{570}$, $\frac{60}{570}$

4. Reduce $\frac{11}{13}$, $\frac{3}{4}$ of $1\frac{1}{4}$, $\frac{9}{11}$ and $\frac{5}{7}$ to a common denominator. *Ans.* $\frac{13552}{16016}$, $\frac{15015}{16016}$, $\frac{13104}{16016}$, $\frac{11440}{16016}$

RULE 2.

To reduce any given fractions to others, which shall have the least common denominator.

1. Find the least common multiple of all the denominators of the given fractions, and it will be the common denominator required.

2. Divide the common denominator by the denominator of each fraction, and multiply the quotient by the numerator, and the products will be the numerators of the fractions required.

EXAMPLES.

1. Reduce $\frac{1}{2}$, $\frac{2}{3}$ and $\frac{5}{6}$ to fractions, having the least common denominator possible.

$$\begin{array}{r|ccc} 3 & 2. & 3. & 6 \\ \hline 2 & 2. & 1. & 2 \\ \hline & 1. & 1. & 1 \end{array}$$

$1 \times 1 \times 1 \times 2 \times 3 = 6 =$ *least common denom.*

$6 \div 2 \times 1 = 3$ *the 1st numerator*; $6 \div 3 \times 2 = 4$ *the 2d numerator*; $6 \div 6 \times 5 = 5$ *the 3d numerator.*

Whence the required fractions are, $\frac{3}{6}$, $\frac{4}{6}$, $\frac{5}{6}$.

2. Reduce

2. Reduce $\frac{7}{12}$ and $\frac{11}{18}$ to fractions having the least common denominator. *Ans.* $\frac{21}{36}$, $\frac{22}{36}$

3. Reduce $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{5}{6}$ to fractions having the least common denominator. *Ans.* $\frac{6}{12}$, $\frac{8}{12}$, $\frac{9}{12}$, $\frac{10}{12}$

4. Reduce $\frac{2}{5}$, $\frac{4}{6}$, $\frac{5}{9}$ and $\frac{7}{10}$ to fractions having the least common denominator. *Ans.* $\frac{36}{90}$, $\frac{60}{90}$, $\frac{50}{90}$, $\frac{63}{90}$

5. Reduce $\frac{1}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{8}$, $\frac{11}{16}$, and $\frac{17}{24}$ to equivalent fractions having the least common denominator possible.
Ans. $\frac{16}{48}$, $\frac{36}{48}$, $\frac{40}{48}$, $\frac{42}{48}$, $\frac{33}{48}$, $\frac{34}{48}$

CASE 7.

To find the value of a fraction in the known parts of the integer.

RULE*.

Multiply the numerator by the parts in the next inferior denomination, and divide the product by the denominator.

And if any thing remains, multiply it by the next inferior denomination, and divide by the denominator as before; and so on as far as necessary; and the quotients, placed in order, will be the answer required.

EXAMPLES.

1. What is the value of $\frac{5}{7}$ of a shilling?

```
    5
   12
   --
 7)60
   --
    8 - 4
        4
        --
     7)16
        --   Ans. 8½d. 2/7
        2 - 2
```

2. What is the value of $\frac{3}{8}$ of a pound sterling? *Ans.* 7*s.* 6*d.*
3. What is the value of $\frac{2}{9}$ of a guinea? *Ans.* 4*s.* 8*d.*
4. What is the value of $\frac{4}{7}$ of half a crown? *Ans.* 1*s.* $5\frac{1}{7}$*d.*

* The numerator of a fraction may be considered as a remainder, and the denominator as a divisor; therefore this rule has its reason in the nature of compound division, and the valuation of remainders in the rule of three, which have been already sufficiently explained.

5. What is the value of $\frac{13}{19}$ of a moidore? *Ans.* 18*s.* $5\frac{13}{19}$*d.*

6. What is the value of $\frac{3}{5}$ of a pound troy?
Ans. 7. *oz.* 4 *dwts.*

7. What is the value of $\frac{4}{7}$ of a pound avoirdupois?
Ans. 9 *oz.* $2\frac{2}{7}$ *dr.*

8. What is the value of $\frac{5}{9}$ of an ell english?
Ans. 2 *qr.* $3\frac{1}{9}$ *na.*

9. What is the value of $\frac{5}{8}$ of an acre? *Ans.* 2 *ro.* 20 *po.*

10. What is the value of $\frac{2}{15}$ of a *hhd.* of ale?
Ans. 7 *gall.* $1\frac{3}{5}$ *pi.*

11. What is the value of $\frac{7}{8}$ of a tun of wine?
Ans. 3 *hhds.* 31 *gall.* 2 *qr.*

12. What is the value of $\frac{7}{9}$ of a *cwt.*?
Ans. 3 *qr.* 3 *lb.* 1 *oz.* $12\frac{4}{9}$ *dr.*

13. What is the value of $\frac{5}{9}$ of a quarter of corn?
Ans. 4 *bu.* 1 *pe.* 1 *ga.* $2\frac{2}{9}$ *qr.*

14. What is the value of $\frac{7}{13}$ of a day?
Ans. 12 *ho.* 55. *min.* $23\frac{1}{13}$ *sec.*

CASE 8.

To reduce a fraction of one denomination to that of another, which shall have the same value.

RULE*.

Consider how many of the less denomination make one of the greater; and multiply the numerator by that number, if the reduction be to a less denomination, or the denominator, if to a greater.

EXAMPLES.

1. Reduce $\frac{5}{6}$ of a penny to the fraction of a pound.

$\frac{5}{6} \times \frac{1}{12} \times \frac{1}{20} = \frac{5}{1440} = \frac{1}{288}$ *the answer.*

And $\frac{1}{288} \times \frac{20}{1} \times \frac{12}{1} = \frac{240}{288} = \frac{5}{6}$ *the proof.*

2. Reduce $\frac{2}{3}$ of a farthing to the fraction of a pound.
Ans. $\frac{1}{1440}$

3. Reduce $\frac{1}{18}$*l.* to the fraction of a penny. *Ans.* $\frac{40}{3}$

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* The reason of this practice is explained in the rule for reducing compound fractions to single ones.

The rule might have been distributed into 2 or 3 different cases, but the directions here given may easily be applied to any question that can be proposed in those cases, and will be more readily understood by an example or two, than by a multiplicity of words. Let there be taken one question in each of the cases.

Reduce

4. Reduce $\frac{4}{5}$ of a *dwt.* to the fraction of a pound troy. *Ans.* $\frac{1}{300}$

5. Reduce $\frac{6}{7}$ of a pound avoirdupois to the fraction of *cwt.* *Ans.* $\frac{3}{392}$.

6. Reduce $\frac{9}{6352}$ of a *hhd.* of wine to the fraction of a pint. *Ans.* $\frac{9}{13}$

7. Reduce $\frac{3}{13}$ of a month to the fraction of a day. *Ans.* $\frac{84}{13}$

8*. Reduce 7*s.* 3*d.* to the fraction of a pound. *Ans.* $\frac{29}{80}$

9. Express 6 *fur.* 16 *po.* in the fraction of a mile. *Ans.* $\frac{4}{5}$

10†. Reduce $\frac{2}{7}l.$ to the fraction of a guinea. *Ans.* $\frac{40}{147}$

11. Express $\frac{5}{8}$ of a crown in the fraction of a guinea. *Ans.* $\frac{25}{168}$

12. Express $\frac{5}{6}$ of half a crown in the fraction of a shilling. *Ans.* $\frac{25}{12}$

13. Express $\frac{6}{7}$ of a moidore in the fraction of a crown. *Ans.* $\frac{162}{35}$

ADDITION OF VULGAR FRACTIONS.

RULE‡.

1. Reduce compound fractions to single ones; mixed numbers to improper fractions; fractions of different denominations to those of the same; and all of them to a common denominator.

2. Add all the numerators together, and place the sum over the common denominator, and it will be the sum of the fractions required.

EXAMPLES.

1. Add $\frac{2}{3}$ and $\frac{3}{4}$ together.

$$\left.\begin{array}{l} 2 \times 4 = 8 \\ 3 \times 3 = 9 \end{array}\right\} \textit{Numerators.}$$

$$3 \times 4 = 12 \textit{ Denominator.}$$

Therefore $\frac{8}{12} + \frac{9}{12} = \frac{17}{12} = 1\frac{5}{12} =$ *sum required.*

2. Add

Thus * 7*s.* 3*d.* = 87*d.* and 1*l.* = 240*d.* ∴ $\frac{87}{240} = \frac{29}{80}$ the answer.

† $\frac{2}{7}l. = \frac{2}{7}$ of $\frac{20}{1} = \frac{2 \times 20}{7 \times 1} = \frac{40}{7}s.$ and $\frac{40}{7}$ of $\frac{1}{21} = \frac{40 \times 1}{7 \times 21}$ $= \frac{40}{147}$ guinea, the answer.

‡ Fractions before they are reduced to a common denominator are entirely dissimilar, and therefore cannot be incorporated with one another; but when they are reduced to a common denominator, and made parts

2. What is the sum of $2\frac{1}{3}$, $\frac{4}{5}$, and $\frac{1}{2}$ of $\frac{3}{4}$?

First $2\frac{1}{3} = \frac{7}{3}$, *and* $\frac{1}{2}$ *of* $\frac{3}{4} = \frac{3}{8}$

$\therefore$ *the fractions are* $\frac{7}{3}$, $\frac{3}{8}$ *and* $\frac{4}{5}$.

$$\left.\begin{array}{l} 7 \times 8 \times 5 = 280 \\ 3 \times 3 \times 5 = 45 \\ 4 \times 3 \times 8 = 96 \end{array}\right\} \textit{Numerators.}$$

$$3 \times 8 \times 5 = 120 \textit{ Denominator.}$$

$$\frac{280 \times 45 \times 96}{120} = \frac{421}{120} = 3\frac{61}{120} \textit{ the answer.}$$

3. Add $\frac{5}{8}$, $7\frac{1}{2}$ and $\frac{1}{3}$ of $\frac{3}{4}$ together. *Ans.* $8\frac{3}{8}$
4. What is the sum of $\frac{3}{5}$, $\frac{4}{5}$ of $\frac{1}{3}$, and $9\frac{3}{20}$? *Ans.* $10\frac{1}{60}$
5. What is the sum of $\frac{9}{10}$ of $6\frac{7}{8}$, $\frac{4}{7}$ of $\frac{1}{2}$, and $7\frac{1}{2}$? *Ans.* $13\frac{102}{112}$
6. Add $\frac{1}{7}$ *l.* $\frac{2}{9}$ *s.* and $\frac{5}{12}$ of a penny together. *Ans.* 3 *s.* $1\frac{1}{4}$ *d.* $\frac{10}{21}$
7. What is the sum of $\frac{2}{7}$ of 15 *l.* $3\frac{3}{7}$ *l.* $\frac{1}{3}$ of $\frac{5}{7}$ of $\frac{3}{5}$ of a *l.* and $\frac{2}{3}$ of $\frac{3}{7}$ of a *s.* *Ans.* 7 *l.* 17 *s.* $5\frac{1}{7}$*d.*
8. Add $\frac{2}{3}$ of a yard, $\frac{3}{4}$ of a foot, and $\frac{3}{8}$ of a mile together. *Ans.* 660 *yds.* $\frac{11}{12}$
9. Add $\frac{1}{3}$ of a week, $\frac{1}{4}$ of a day, and $\frac{1}{2}$ of an hour together. *Ans.* 2 *da.* $14\frac{1}{2}$ *ho.*
10. Required the sum of 4, $3\frac{1}{5}$, and $\frac{2}{3}$ of $\frac{3}{4}$.
11. Required the sum of $\frac{5}{9}$ of a guinea, and $\frac{3}{8}$ of a moidore.
12. What is the sum of $\frac{4}{7}$ of a *cwt.* $8\frac{5}{6}$ *lb.* and $3\frac{9}{10}$ ounces?
13. What is the sum of $3\frac{1}{2}$ English ells, $4\frac{1}{2}$ yards, and $\frac{5}{7}$ of a nail?
14. What is the sum of $\frac{3}{4}$ of a *hhd.* of ale, $2\frac{5}{7}$ *gallons,* and $\frac{3}{4}$ of $\frac{4}{5}$ of a pint?

SUBTRACTION OF VULGAR FRACTIONS.

RULE.

Prepare the fractions as in addition, and the difference of the numerators, written above the common denominator, will give the difference of the fractions required.

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parts of the same thing, their sum, or difference, may then be as properly expressed by the sum or difference of the numerators, as the sum or difference of any two quantities whatever, by the sum or difference of their individuals; whence the reason of the rules, both for addition and subtraction, is manifest.

EXAM-

EXAMPLES.

1. What is the difference of $\frac{3}{4}$ and $\frac{5}{7}$?

$$\left.\begin{array}{l} 3 \times 7 = 21 \\ 5 \times 4 = 20 \end{array}\right\} \textit{Numerators.}$$

$$4 \times 7 = 28 \textit{ Denominator.}$$

$$\textit{Therefore } \frac{21 - 20}{28} = \frac{1}{28} \textit{ the answer.}$$

2. What is the difference between $\frac{2}{3}$, and $\frac{2}{9}$ of $\frac{3}{7}$?

$$\frac{2}{9} \textit{ of } \frac{3}{7} = \frac{6}{63} = \frac{2}{21}, \ \& \ \frac{2}{3} = \frac{14}{21}$$

$$\textit{Therefore } \frac{14}{21} - \frac{2}{21} = \frac{12}{21} = \frac{4}{7} \textit{ the answer required.}$$

3. From $\frac{97}{100}$ take $\frac{3}{7}$. *Ans.* $\frac{379}{700}$
4. From $69\frac{1}{3}$ take $14\frac{3}{7}$. *Ans.* $81\frac{19}{21}$
5. From $14\frac{1}{4}$ take $\frac{2}{3}$ of 19. *Ans.* $1\frac{7}{12}$
6. From $\frac{1}{2}$ *l.* take $\frac{3}{4}$ *s.* *Ans.* 9 *s.* 3 *d.*
7. From $\frac{3}{5}$ *oz.* take $\frac{7}{8}$ *dwt.* *Ans.* 11 *dwts.* 3 *gr.*
8. From $\frac{2}{3}$ of a league take $\frac{7}{10}$ of a mile. *Ans.* 1 *mi.* 2 *fur.* 16 *po.*
9. From 7 weeks take $9\frac{7}{10}$ days. *Ans.* 5 *we.* 4 *da.* 7 *ho.* 12 *min.*
10. From $4\frac{3}{7}$ of a hundred weight take $14\frac{9}{10}$ *lb.*
11. What is the difference of $100\frac{5}{7}$ and $\frac{3}{4}$ of 10?
12. What is the difference of 18, and $\frac{4}{7}$ of $\frac{9}{10}$?

MULTIPLICATION OF VULGAR FRACTIONS.

RULE*.

Reduce compound fractions to simple ones, and mixed numbers to improper fractions; then multiply the numerators together for a numerator, and the denominators for a denominator, and it will give the product required.

* Multiplication by a fraction implies the taking some part or parts of the multiplicand, and therefore, may be truly expressed by a compound fraction. Thus $\frac{3}{4}$ multiplied by $\frac{5}{8}$, is the same as $\frac{3}{4}$ of $\frac{5}{8}$; and as the directions of the rule agree with the method already given to reduce these fractions to simple ones, it is shewn to be right.

EXAMPLES.

1. Required the product of $\frac{4}{5}$ and $\frac{7}{8}$.

$$\frac{4 \times 7}{5 \times 8} = \frac{28}{40} = \frac{14}{20} = \frac{7}{10} \text{ the answer.}$$

2. Required the continued product of $2\frac{1}{2}$, $\frac{1}{8}$, $\frac{1}{3}$ of $\frac{5}{6}$, and 2.

$$2\tfrac{1}{2}, = \tfrac{5}{2}, \tfrac{1}{3} \text{ of } \tfrac{5}{6} = \frac{1 \times 5}{3 \times 6} = \tfrac{5}{18}, \text{ and } 2 = \tfrac{2}{1};$$

$$\text{Then } \tfrac{5}{2} \times \tfrac{1}{8} \times \tfrac{5}{18} \times \tfrac{2}{1} = \frac{5 \times 1 \times 5 \times 2}{2 \times 8 \times 18 \times 1} = \tfrac{25}{144}$$

the answer.

3. Multiply $\frac{4}{15}$ by $\frac{5}{24}$. *Ans.* $\frac{1}{18}$
4. Multiply $4\frac{1}{2}$ by $\frac{1}{8}$. *Ans.* $\frac{9}{16}$
5. Multiply $\frac{1}{2}$ of 7 by $\frac{1}{2}$. *Ans.* $1\frac{3}{4}$
6. Multiply $\frac{2}{9}$ of $\frac{3}{5}$ by $\frac{5}{8}$ of $3\frac{2}{7}$. *Ans.* $\frac{23}{84}$
7. Multiply $4\frac{1}{2}$, $\frac{3}{4}$ of $\frac{1}{7}$, and $18\frac{4}{5}$ continually together. *Ans.* $9\frac{9}{140}$
8. What is the continued product of $\frac{2}{3}$, $3\frac{1}{4}$, 5, and $\frac{3}{4}$ of $\frac{3}{5}$? *Ans.* $4\frac{7}{8}$
9. What is the continued product of 5, $\frac{2}{3}$, $\frac{2}{7}$ of $\frac{3}{5}$, and $4\frac{1}{6}$?
10. What is the continued product of 14, $\frac{5}{6}$, $\frac{4}{5}$ of 9, and $6\frac{2}{7}$?

DIVISION OF VULGAR FRACTIONS.

RULE*.

Prepare the fractions as before; then invert the divisor, and proceed exactly as in multiplication.

* The reason of the rule may be shewn thus. Suppose it were required to divide $\frac{3}{4}$ by $\frac{2}{5}$. Now $\frac{3}{4} \div 2$ is manifestly $\frac{1}{2}$ of $\frac{3}{4}$ or $\frac{3}{4 \times 2}$; but $\frac{2}{5} = \frac{1}{5}$ of 2, $\therefore \frac{1}{5}$ of 2, or $\frac{2}{5}$ must be contained 5 times as often in $\frac{3}{4}$ as 2 is; that is $\frac{3 \times 5}{4 \times 2}$ = the answer; which is according to the rule; and will be so in all cases.

Note, A fraction is multiplied by an integer, by dividing the denominator by it, or multiplying the numerator. And divided by an integer, by dividing the numerator, or multiplying the denominator.

EXAM-

EXAMPLES.

1. It is required to divide $\frac{4}{7}$ by $\frac{3}{5}$.

$$\frac{4}{7} \div \frac{3}{5} = \frac{4}{7} \times \frac{5}{3} = \frac{20}{21} \textit{ answer.}$$

2. Divide $\frac{1}{5}$ of 19 by $\frac{2}{3}$ of $\frac{3}{4}$.

$$\tfrac{1}{5} \text{ of } 19 = \frac{1 \times 19}{5 \times 1} = \tfrac{19}{5}, \textit{ and } \tfrac{2}{3} \textit{ of } \tfrac{3}{4} = \frac{2 \times 3}{3 \times 4} = \frac{6}{12} = \tfrac{1}{2}$$

$$\therefore \tfrac{19}{5} \times \tfrac{2}{1} = \frac{19 \times 2}{5 \times 1} = \tfrac{38}{5} = 7\tfrac{3}{5} \textit{ the quotient required.}$$

3. Divide $\frac{4}{7}$ by $\frac{2}{3}$. *Ans.* $\frac{6}{7}$
4. Divide $9\frac{1}{6}$ by $\frac{1}{2}$ of 7. *Ans.* $2\frac{13}{21}$
5. Divide $3\frac{1}{6}$ by $9\frac{1}{2}$. *Ans.* $\frac{1}{3}$
6. Let $\frac{7}{8}$ be divided by 4. *Ans.* $\frac{7}{32}$
7. Let $\frac{1}{2}$ of $\frac{2}{3}$ be divided by $\frac{2}{3}$ of $\frac{3}{4}$. *Ans.* $\frac{2}{3}$
8. Let 5 be divided by $\frac{7}{10}$. *Ans.* $7\frac{1}{7}$
9. Let $5205\frac{1}{5}$ be divided by $\frac{4}{5}$ of 91. *Ans.* $71\frac{1}{2}$
10. Required the quotient of 100 divided by $4\frac{7}{8}$.
11. Required the quotient of $\frac{3}{4}$ of $\frac{7}{8}$ divided by $\frac{2}{3}$.
12. Required the quotient of $\frac{5}{6}$ of 50 divided by $4\frac{1}{3}$.

RULE OF THREE DIRECT IN VULGAR FRACTIONS.

RULE*.

Make the neceſſary preparations as before directed, and invert the firſt term of the proportion; then multiply the three terms continually together, and the product will be the anſwer.

EXAMPLES.

1. If $\frac{3}{5}$ of a yard coſt $\frac{7}{12}$ of a *l.* what will $\frac{6}{15}$ of an Engliſh ell coſt?

$$\textit{First } \tfrac{3}{5} \textit{ of a yard} = \tfrac{3}{5} \textit{ of } \tfrac{4}{1} \textit{ of } \tfrac{1}{5} = \frac{3 \times 4 \times 1}{5 \times 1 \times 5} = \tfrac{12}{25} \textit{ of an ell.}$$

Then $\frac{12}{25}$ *ell* : $\frac{7}{12}$ *l.* :: $\frac{6}{15}$ *ell.*

5

$$\textit{And } \frac{7}{12} \times \frac{6}{15} \times \frac{25}{12} = \frac{7 \times 6 \times 25}{12 \times 15 \times 12} = \frac{35}{72}.$$

$$= 9s.\ 8d.\ \tfrac{2}{3} \textit{ the answer.}$$

* This rule depends upon the ſame principles as the rule of three in whole numbers.

2. If $\frac{3}{5}$ of an ell of holland cost $\frac{1}{3}$ *l.* what will 12 $\frac{2}{3}$ ells cost?
Ans. 7 *l.* 0 *s.* 8 $\frac{3}{4}$ *d.*

3. If $\frac{5}{7}$ *oz.* cost $\frac{11}{12}$ *l.* what will 1 *oz.* cost? *Ans.* 1 *l.* 5 *s.* 8 *d.*

4. If $\frac{3}{16}$ of a ship cost 273 *l.* 2 *s.* 6 *d.* what is $\frac{5}{32}$ of her worth?
Ans. 227 *l.* 12 *s.* 1 *d.*

5. At 1 $\frac{1}{2}$ *l. per cwt.* what does 3 $\frac{1}{3}$ *lb.* come to? *Ans.* 10 $\frac{1}{2}$ *d.* $\frac{6}{7}$

6. If $\frac{5}{8}$ of a gallon of wine cost $\frac{5}{8}$ *l.* what will $\frac{5}{9}$ of a tun cost?
Ans. 140 *l.*

7. A mercer bought 3 $\frac{1}{2}$ pieces of silk, each containing 24 $\frac{1}{3}$ yards, at 6 *s.* $\frac{1}{2}$ *d. per yard*, what does the whole come to?
Ans. 25 *l.* 14 *s.* 6 $\frac{1}{2}$ *d.* $\frac{1}{3}$

8. Agreed for the carriage of 2 $\frac{1}{2}$ tons of goods 2 $\frac{9}{10}$ miles for $\frac{3}{40}$ of a guinea, what is that *per cwt.* for a mile?
Ans. $\frac{378}{725}$ *of a farthing.*

9. A person having $\frac{3}{5}$ of a coal mine, sells $\frac{3}{4}$ of his share for 171 *l.* what is the whole mine worth? *Ans.* 380 *l.*

10. If $\frac{5}{8}$ of a *cwt.* cost 4 $\frac{7}{9}$ *l.* what will 4 $\frac{1}{2}$ *lb.* cost?

RULE OF THREE INVERSE IN VULGAR FRACTIONS.

RULE.

Prepare the fractions, as in the former rules, and invert the third term of the proportion; then multiply the three terms continually together, and the product will be the answer.

EXAMPLES.

1. What quantity of shalloon that is $\frac{3}{4}$ *yd.* wide, will line 9 $\frac{1}{2}$ yards of cloth that is 2 $\frac{1}{2}$ yards wide?

First 2 $\frac{1}{2}$ *yds.* $= \frac{5}{2}$, & 9 $\frac{1}{2}$ *yds.* $= \frac{19}{2}$.

Then $\frac{5}{2}$ *yds.* : $\frac{19}{2}$ *yds.* : $\frac{3}{4}$ *yd.*

$$\textit{And } \frac{5}{2} \times \frac{19}{2} \times \frac{4}{3} = \frac{5 \times 19 \times 4}{2 \times 2 \times 3} = \frac{95}{3}$$

$= 31\frac{2}{3}$ *yds. the answer.*

2. How much in length that is 7 $\frac{7}{9}$ inches broad will make a foot square? *Ans.* 18 $\frac{18}{35}$ *inches.*

3. How much in length that is 11 $\frac{11}{12}$ poles broad will make a square acre? *Ans.* 13 $\frac{61}{143}$ *po.*

4. If

4. If when wheat is 5 *s. per* bushel, the penny-loaf weighs $6\frac{9}{10}$ *oz.* what ought it to weigh when wheat is 8 *s.* 6 *d. per* bushel? *Ans.* $4\frac{1}{17}$ *oz.*

5. If when the days are $13\frac{5}{8}$ hours long, a traveller performs his journey in $35\frac{1}{2}$ days, in how many days will he perform the same journey when the days are $11\frac{9}{10}$ hours long? *Ans.* $40\frac{615}{952}$ *days.*

6. How many yards of ell wide flannel are sufficient to line a cloak, containing $18\frac{7}{8}$ *yds.* of camblet $\frac{3}{4}$ yard wide? *Ans.* 11 *yds.* 1 *qr.* $1\frac{1}{5}$ *na.*

7. A regiment of soldiers consisting of 976 men, are to be new clothed, each coat to contain $2\frac{1}{2}$ yards of cloth that is $1\frac{5}{8}$ *yd.* wide, and lined with shalloon $\frac{7}{8}$ *yd.* wide; how many yards of shalloon will line them? *Ans.* 4531 *yds.* 1 *qr.* $2\frac{6}{7}$ *na.*

8. If a coat and waistcoat can be made of $3\frac{3}{4}$ *yds.* of broad cloth of $1\frac{1}{2}$ *yds.* in breadth, how many yards of stuff of $\frac{5}{8}$ *yds.* in breadth will it require to fit the same person? *Ans.* 9 *yds.*

DECIMAL FRACTIONS.

A DECIMAL FRACTION is that whose denominator is an unit with as many cyphers annexed as the numerator has places; and is usually expressed by writing the numerator only, with a point before it, on the left hand: thus, $\frac{5}{10}$, $\frac{25}{100}$, $\frac{75}{1000}$, $\frac{123}{100000}$, &c. are decimal fractions, and are expressed by .5 .25 .075 and .00123 respectively.

The 1*st.* 2*d.* 3*d.* 4*th*, &*c.* places of decimals, counting from the left hand towards the right, are called primes, seconds, thirds, fourths, &c.

Cyphers to the right hand of decimals make no alteration in their value; for .5 .50 .500, &c. are decimals, having the same value, being each $=\frac{1}{2}$; but if they are placed on the left hand, they decrease their value in a ten-fold proportion. Thus, .5, .05, .005, &c. are 5 tenth parts, 5 hundredth parts, 5 thousandth parts, &c. respectively*.

* As in notation of whole numbers the values of the figures increase in a ten-fold proportion, from the right hand to the left; so in decimals, their values decrease in the same ten-fold proportion, from the left hand to the right. Thus, .5 expresses 5 tenth parts of the integer, .05, 5 hundredth parts, &c.

ADDI-

ADDITION OF DECIMALS.

RULE.

1. Place the numbers under each other according to the value of their places.

2. Find their sum as in whole numbers, and point off as many places, for decimals, as are equal to the greatest number of decimal places in any of the given numbers.

EXAMPLES.

1. Find the sum of 25.074 + 1.8254 + 125 + .0567876 + 1776.111.

```
  25.074
   1.8254
 125
    .0567876
1776.111
------------
1928.0671876 the sum.
```

2. Find the sum of 376.25 + 86.125 + 637.4725 + 6.5 + 358.865 + 41.02 *Anſ.* 1506.2325
3. Required the sum of 3.5 + 47.25 + 927.01 + 2.0073 + 1.5. *Anſ.* 981.2673
4. Required the sum of 276 + 54.321 + .65 + 112 + 12.5 + .0463. *Anſ.* 455.5173

SUBTRACTION OF DECIMALS.

RULE.

Place the numbers according to their value; then ſubtract as in whole numbers, and point off the decimals as in addition.

EXAMPLES.

1. Find the difference of 2464.21 and 327.07643.

```
2464.21
 327.07643
----------
2137.13357 the difference.
```

2. From 127.62 take 13.725. *Anſ.* 113.895.
3. From 6213.725 take 162.25. *Anſ.* 6051.475
4. From 3760.279 take 423.0076. *Anſ.* 3337.2714

MULTI-

MULTIPLICATION OF DECIMALS.

RULE*.

1. Place the factors, and multiply them as in whole numbers.

2. Point off as many figures from the product as there are decimal places in both the factors; and if there are not so many places in the product, supply the defect by prefixing cyphers.

EXAMPLES.

```
Multiply .02534
      by .03256
         -------
          15204
         12670
         5068
        7602
    -----------
   .0008250704 the product.
```

2. Multiply 79.347 by 23.15. *Ans.* 1836 88305
3. Multiply .63478 by .8204. *Ans.* .520773512
4. Multiply .385746 by .00464. *Ans.* .00178986144

CASE 2.

To contract the operation, so as to retain as many decimal places in the product as may be thought necessary.

RULE.

1. Write the units place of the multiplier under that figure of the multiplicand whose place you would reserve in the product; and dispose of the rest of the figures in a contrary order to what they are usually placed in.

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* To prove the truth of the rule, let .9776 and .823 be the numbers to be multiplied; now these are equivalent to $\frac{9776}{10000}$ and $\frac{823}{1000}$; whence $\frac{9776}{10000} \times \frac{823}{1000} = \frac{8045648}{10000000} = .8045648$ by the nature of notation, which consists of as many places as there are cyphers; that is, of as many places as are in both the numbers; and the same is true of any two numbers whatever.

2. In

2. In multiplying, reject all the figures that are to the right hand of the multiplying digit, and set down the products, so that their right hand figures may fall in a straight line below each other; observing to increase the first figure of every line with what would arise by carrying 1 from 5 to 15, 2 from 15 to 25, &c. from the preceding figures when you begin to multiply, and the sum is the product required.

EXAMPLES.

1. It is required to multiply 27.14986 by 92.41035, and to retain only four places of decimals in the product.

Contracted.	*Common way.*	
27.14986	27.14986	
53014.29	92.41035	
24434874	13	574930
592997	81	44958
108599	2714	986
2715	108599	44
81	542997	2
14	24434874	
2508.9280	2508.9280	650510

2. Multiply 245.378263 by 72.4385, reserving 5 places of decimals in the product. *Ans.* 17774.83330
3. Multiply .248264 by .725234, reserving 6 figures, 5 figures and 4 figures in the product respectively.
Ans. .180049. 18005, *and* .1800
4. Multiply 8634.875 by 843.7527, reserving only the integers in the product. *Ans.* 7285699

DIVISION OF DECIMALS.

RULE*.

1. Divide as in whole numbers, and from the right hand of the quotient point off as many places for decimals as the decimal places in the dividend exceed those in the divisor.

* The reason of pointing off as many decimal places in the quotient as those in the dividend exceed the divisor, will easily appear; for since the number of decimal places in the dividend is equal to those in the divisor and quotient taken together, by the nature of multiplication; it therefore follows that the quotient contains as many as the dividend exceeds the divisor.

2. If

2. If the places of the quotient are not so many as the rule requires, supply the defect by prefixing cyphers.

3. If at any time there be a remainder, or the decimal places in the divisor be more than those in the dividend, cyphers may be affixed to the dividend, and the quotient carried on to any degree of exactness.

EXAMPLES.

```
179).48624097(.00271643   2685)27.0000(100.55865
      1282                     15000
       294                      15750
       1150                      23250
         769                      17700
          537                      15900
          000                       24750
         ----                      -----
          &c.                        &c.
```

1. Divide 14 by .7854. *Ans.* 17.825 &c.
2. Divide 234.70525 by 64.25. *Ans.* 3.653
3. Divide 217.568 by 100. *Ans.* 2.17568
4. Divide .8727587 by .162. *Ans.* 5.38739 &c.

CASE 2.

To contract the operation, so as to retain as many decimal places in the quotient as may be thought necessary.

RULE.

1. Take as many of the left hand figures of the divisor as will be equal to the number of integers and decimals in the quotient, and find how many times they may be had in the first figures of the dividend, as usual.

2. Let each remainder be a new dividend; and for every such dividend, leave out one figure to the right hand of the divisor, remembering to carry for the increase of the figures cut off, as in the second rule of multiplication.

Note. When there are not so many figures in the divisor as are required to be in the quotient, begin the operation with all the figures, as usual, and continue it till the number of figures in the divisor, and those remaining to be found in the quotient be equal, after which use the contraction.

EXAMPLES.

1. Divide 2508.928065051 by 92.41035, so as to have 4 places of decimals in the quotient.

Contracted way.

92.41035)2508.928065051(27.1498
660721
13849
4608
912
80
6

Common way.

92.41035)2508.928065051(27.1498

660721	06
13848	651
4607	5800
911	16605
79	472901
5	544621

2. Divide 721.17562 by 2.257432, and let there be only 3 places of decimals in the quotient. *Ans.* 319.467
3. Divide 12.169825 by 3.14159, and preserve 5 places of decimals in the quotient. *Ans.* 3.87377
4. Divide 87.076326 by 9.365407, and let there be 7 places of decimals in the quotient. *Ans.* 9.2976554

REDUCTION OF DECIMALS.

CASE I.

To reduce a vulgar fraction to its equivalent decimal one.

RULE*.

Divide the numerator by the denominator, and the quotient will be the decimal required.

EXAM-

* Let the given vulgar fraction, whose decimal expression is required, be $\frac{7}{13}$. Now since every decimal fraction has 10, 100, or 1000, &c. for its denominator; and, if two fractions are equal, it will be, as the denominator of one is to its numerator, so is the denominator of the other to its numerator; therefore 13 : 7 :: 1000, &c. : $\frac{7 \times 1000\ \&c.}{13} = \frac{7000}{13}$, &c. = .53846 the numerator of the decimal required; and is the same as by the rule.

The

EXAMPLES.

1. Reduce $\frac{5}{24}$ to a decimal.

$$4)5.000000$$
$$6)1.250000$$
$$.208333, \&c.$$

2. Required the equivalent decimal expressions for $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{3}{4}$. *Ans.* .25 .5 *and* .75
3. What is the decimal of $\frac{3}{8}$? *Ans.* .375
4. What is the decimal of $\frac{1}{25}$? *Ans.* .04
5. What is the decimal of $\frac{3}{192}$? *Ans.* .015625
6. Let $\frac{275}{3842}$ be expressed decimally. *Ans.* .071577, &c.

CASE 2.

To reduce numbers of different denominations to their equivalent decimal values.

RULE*.

1. Write the given numbers perpendicularly under each other, for dividends, proceeding orderly from the least to the greatest.

2. Oppo-

The following method of throwing a vulgar fraction, whose denominator is a prime number, into a decimal consisting of a great number of figures, is given by Mr. *Colson* in page 162 of *Sir Isaac Newton's Fluxions*.

EXAMPLE.

Let $\frac{1}{29}$ be the fraction which is to be converted into an equivalent decimal.

Then, by dividing in the common way till the remainder becomes a single figure, we shall have $\frac{1}{29} = .03448\,\frac{8}{29}$ for the complete quotient, and this equation being multiplied by the numerator 8, will give $\frac{8}{29} = .27584\,\frac{64}{29}$, or rather $\frac{8}{29} = .27586\,\frac{6}{29}$: and if this be substituted instead of the fraction in the first equation, it will make $\frac{1}{29} = .0344827586\,\frac{6}{29}$. Again, let this equation be multiplied by 6, and it will give $\frac{6}{29} = .2068965517\,\frac{7}{29}$; and then by substituting as before $\frac{1}{29} = .03448275862068965517\,\frac{7}{29}$; and so on as far as may be thought proper.

* The reason of the rule may be explained from the first example: thus, three farthings is $\frac{3}{4}$ of a penny, which brought to a decimal is .75;

conse-

2. Opposite to each dividend, on the left hand, place such a number for a divisor as will bring it to the next superior name, and draw a line between them.

3. Begin with the highest, and write the quotient of each division, as decimal parts, on the right hand of the dividend next below it; and so on till they are all used, and the last quotient will be the decimal sought.

EXAMPLES.

1. Reduce 15*s.* 9*d.* $\frac{3}{4}$ to the decimal of a pound.

4	3.
12	9.75
20	15.8125

.790625 *the decimal required.*

2. Reduce 9*s.* to the decimal of a pound. *Ans.* .45.
3. Reduce 19*s.* 5$\frac{1}{2}$*d.* to the decimal of a pound. *Ans.* .972916
4. Reduce 10*oz.* 18*dwt.* 16*grs.* to the decimal of a *lb.* troy. *Ans.* .911111, &c.
5. Reduce 2 *qrs.* 14 *lb.* to the decimal of a *cwt.* *Ans.* .625, &c.
6. Reduce 17*yds.* 1 *ft.* 6 *in.* to the decimal of a mile. *Ans.* .00994318, &c.
7. Reduce 3 *qrs.* 2 *na.* to the decimal of a yard. *Ans.* .875
8. Reduce 1 *ro.* 14 *po* to the decimal of an acre. *Ans.* .3375
9. Reduce 1 *gall.* of wine to the decimal of a *hhd.* *Ans.* .015873
10. Reduce 3 *bu.* 1 *pe.* to the decimal of a quarter. *Ans* .40625
11. Reduce 10 *we.* 2 *da.* to the decimal of a year. *Ans.* .1972602, &c.

CASE 3.

To find the decimal of any number of shillings, pence and farthings by inspection.

consequently 9$\frac{3}{4}$ *d.* may be expressed 9.75 *d.*; but 9.75 is $\frac{975}{100}$ of a penny = $\frac{975}{1200}$ of a shilling, which brought to a decimal is .8125; and, therefore 15*s.* 9$\frac{3}{4}$ *d.* may be expressed 15.8125*s.* In like manner 15.8125*s.* is $\frac{158125}{100000}$ of a shilling = $\frac{158125}{1000000}$ of a pound, =, by bringing it to a decimal, to .790625*l.* as by the rule.

RULE.

RULE*.

Write half the greatest even number of shillings for the first decimal figure, and let the farthings in the given pence and farthings possess the second and third places; observing to increase the second place by 5, if the shillings are odd, and the third place by 1, when the farthings exceed 12, and by 2 when they exceed 37.

EXAMPLES.

1. Find the decimal of 15*s.* $8\frac{1}{2}$*d.* by inspection.

$$\begin{array}{ll} .7 & = \tfrac{1}{2} \textit{ of } 14s. \\ \;\;5 & \textit{for the odd shilling.} \\ \;\;34 & = \textit{farthings in } 8\tfrac{1}{2}d. \\ \;\;\;\;1 & \textit{for the excess of } 12. \\ \hline .785 & = \textit{decimal required.} \end{array}$$

2. Find by inspection the decimal expressions of 16*s.* $4\frac{1}{2}$*d.* and 13*s.* $10\frac{1}{2}$*d.* *Ans.* .819 *and* .694

3. Value the following sums by inspection, and find their total, *viz.* 19*s.* $11\frac{1}{4}$*d.* + 6*s.* 2*d.* + 12*s.* $8\frac{3}{4}$*d.* + 1*s.* $10\frac{1}{4}$*d.* + $\frac{3}{4}$*d.* + $1\frac{1}{4}$*d.* *Ans.* 2.043 *the total.*

CASE 4.

To find the value of any given decimal in terms of the integer.

RULE.

1. Multiply the decimal by the number of parts in the next less denomination, and cut off as many places for a remainder, to the right hand, as there are places in the given decimal.

2. Multiply the remainder by the parts in the next inferior denomination, and cut off for a remainder as before.

3. Proceed

* The invention of the rule is as follows: As shillings are so many 20ths of a pound, half of them must be so many 10ths, and consequently take the place of 10ths, in the decimal; but when they are odd, their half will always consist of 2 figures, the first of which will be half the even number, next less, and the second a 5; and this confirms the rule as far as it respects shillings.

Again, farthings are so many 960ths. of a pound; and had it happened that 1000, instead of 960, had made a pound, it is plain any number of farthings

3. Proceed in this manner through all the parts of the integer, and the ſeveral denominations, ſtanding on the left hand, make the anſwer.

EXAMPLES.

1. Find the value of .37623 of a pound.

```
      20
  -------
 7.52460
      12
  -------
 6.29520
       4
  -------
 1.18080 Anſ. 7 s. 6¼ d.
```

2. What is the value of .625 ſhillings? *Anſ.* $7\frac{1}{2}$ *d.*
3. What is the value of .8322916 *l.*? *Anſ.* 16 *s.* $7\frac{1}{2}$ *d.*
4. What is the value of .6725 *cwt.*? *Anſ.* 2 *qrs.* 19 *lb.* 5 *oz.*
5. What is the value of .67 of a league? *Anſ.* 2 *mi.* 0 *fur.* 3 *po.* 1 *yd.* 3 *in.* 1 *bar.*
6. What is the value of .61 of a tun of wine? *Anſ.* 2 *hhds.* 27 *gall.* 2 *qr.* 1 *pi.*
7. What is the value of .461 of a chaldron of coals? *Anſ.* 16 *bu.* 2 *pe.*
8. What is the value of .42857 of a month? *Anſ.* 1 *we.* 4 *da.* 23 *ho.* 59 *min.* 56 *ſe.*

CASE 5.

To find the value of any decimal of a pound by inſpection.

RULE.

Double the firſt figure, or place of tenths, for ſhillings, and if the ſecond be 5, or more than 5, reckon another ſhilling; then call the figures in the ſecond and third places,

farthings would have made ſo many thouſandths, and might have taken their place in the decimal accordingly. But 960 increaſed by $\frac{1}{24}$ part of itſelf, is = 1000; conſequently any number of farthings, increaſed by their $\frac{1}{24}$ part, will be an exact decimal expreſſion for them. Whence if the number of farthings be more than 12, a $\frac{1}{24}$ part is greater than $\frac{1}{2}$, and therefore 1 muſt be added; and when the number of farthings is more than 37, a $\frac{1}{24}$ part is greater than 1 *d.* $\frac{1}{2}$, for which 2 muſt be added; and thus the rule is ſhewn to be right.

after

after 5 is deducted, so many farthings, abating 1 when they are above 12, and 2 when above 37, and the result is the answer.

EXAMPLES.

1. Find the value of .785*l.* by inspection.

   14*s.* . . . = *double of* 7.
   1*s.* . . . *for the* 5 *in the place of tenths.*
   $8\frac{3}{4}$*d.* = 35 *farthings.*
   $\frac{1}{4}$ *for the excess of* 12, *abated.*

   15*s.* $8\frac{1}{2}$*d.* *the answer.*

2. Find the value of .875*l.* by inspection. *Ans.* 17*s.* 6*d.*
3. Value the following decimals by inspection, and find their sum, *viz.* .927*l.* + .351*l.* + .203*l.* + .c61*l.* + .020*l.* + .009*l.* *Ans.* 1*l.* 11*s.* $5\frac{3}{4}$*d.*

RULE OF THREE IN DECIMALS.

EXAMPLES.

1. If $\frac{3}{8}$ of a yard of cloth cost $\frac{2}{5}$ of a pound, what will $\frac{1}{4}$ of an English ell cost?

   $\frac{3}{8}$ = .375
   $\frac{2}{5}$ = .4*l.*
   $\frac{1}{4}$ *ell* = $\frac{5}{16}$*yd.* = .3125
   .375 *yd.* : .4*l.* : : .3125 *yd.*
   .3125

   .375).12500(.333, &c. = 6*s.* 8*d.* *the answer.*
   1125

   1250
   1125

   1250
   1125

   125

2. If an *oz.* of silver cost 5*s.* 6*d.* what is the price of a tankard that weighs 1 *lb.* 10 *oz.* 10 *dwts.* 4 *grs.*?

   Ans. 6*l.* 3*s.* $9\frac{1}{2}$*d.*

3. If

3. If I buy 14 yards of cloth for 10 guineas, how many ells Flemish can I buy for 283*l.* 17*s.* 6*d.* at the same rate?

Ans. 504 *ells* 2 *qrs.*

4. How many *Eng. ells* of Holland may be bought for 25*l.* 18*s.* $1\frac{3}{4}$*d.* at 7*s.* $9\frac{1}{2}$ *d. per* yard? *Ans.* 53 *Eng. ells* 1 *qr.*

CIRCULATING DECIMALS.

CIRCULATING DECIMALS are produced from vulgar fractions whose denominators do not measure their numerators, and are distinguished by the continual repetition of the same figures.

1. The circulating figures are called *repetends*; and if one figure only repeats, it is called a *single repetend*; as .1111, &c.; .3333, &c.

2. A *compound repetend* hath the same figures circulating alternately; as .010101, &c.; .123123123, &c.

3. If other figures arise before those that circulate, the decimal is called a *mixed repetend*; thus, .283333, &c. is a *mixed single repetend*, and .57321321, &c. a *mixed compound repetend*.

4. A single repetend is expressed by writing only the circulating figure with a point over it: thus, .1111, &c. is denoted by $.\dot{1}$, and .333, &c. by $.\dot{3}$.

5. Compound repetends are distinguished by putting a point over the first and last repeating figure; thus .0101, &c. is written $.\dot{0}\dot{1}$, and .123123, &c. $.\dot{1}2\dot{3}$.

6. *Similar circulating decimals* are such as consist of the same number of figures, and begin at the same place, either before or after the decimal point; thus, $.\dot{2}$ and $.\dot{3}$ are similar circulates; as are also $2.\dot{3}\dot{4}$ and $3.\dot{7}\dot{6}$, &c.

7. *Dissimilar repetends* consists of an unequal number of figures, and begin at different places.

8. *Similar and conterminous circulates* are such as begin and end at the same place; as $56.7\dot{8}98\dot{4}$, $8.5\dot{2}68\dot{3}$, and $.05\dot{6}7\dot{8}$, &c.

9. Any finite decimal may be considered as infinite, by annexing cyphers continually to the right-hand of the numerator; thus, $.34 = 34000$, &c. $= .34\dot{0}$.

And any pure circulate may be considered as mixed, by taking the given repetend for a finite part, and making the same repetend a circulate for the infinite part; thus, $.\dot{3}\dot{4} = .34 + .00\dot{3}\dot{4}$.

REDUC-

Reduction of Circulating Decimals.

Case 1.

To reduce a simple repetend to its equivalent vulgar fraction.

Rule*.

1. Make the given decimal the numerator, and let the denominator be a number consisting of as many nines as there are recurring places in the repetend.

2. If there are integral figures in the circulate, as many cyphers must be annexed to the numerator as the highest place of the repetend is distant from the decimal point.

Examples.

1. Required the least vulgar fractions equal to $\dot{6}$ and $\dot{1}2\dot{3}$.

Ans. $.\dot{6} = \frac{6}{9} = \frac{2}{3}$; *and* $.\dot{1}2\dot{3} = \frac{123}{999} = \frac{41}{333}$

2. Reduce $.\dot{3}$ to its equivalent vulgar fraction. *Ans.* $\frac{1}{3}$

3. Reduce $\dot{1}.6\dot{2}$ to its equivalent vulgar fraction. *Ans.* $\frac{1620}{999}$

4. Required the least vulgar fraction equal to $.\dot{7}6923\dot{0}$.

Ans. $\frac{10}{13}$

Case 2.

To reduce a mixed repetend to its equivalent vulgar fraction.

Rule †.

1. To as many nines as there are figures in the repetend, annex as many cyphers as there are finite places, for a denominator.

---

* If unity, with cyphers annexed, be divided by 9 *ad infinitum*, the quotient will be 1 continually; *i. e.* if $\frac{1}{9}$ be reduced to a decimal, it will produce the circulate $.\dot{1}$; and since $.\dot{1}$ is the decimal equivalent to $\frac{1}{9}$, $.\dot{2}$ will $= \frac{2}{9}$, $\dot{3} = \frac{3}{9}$, and so on till $\dot{9} = \frac{9}{9} = 1$.

Therefore every single repetend is equal to a vulgar fraction, whose numerator is the repeating figure, and denominator 9.

Again, $\frac{1}{99}$, and $\frac{1}{999}$, being reduced to decimals, make .010101, &c. and .001001, &c. *ad infinitum*, $= .\dot{0}\dot{1}$ and $.\dot{0}0\dot{1}$; that is $\frac{1}{99} = .\dot{0}\dot{1}$ and $\frac{1}{999} = .\dot{0}0\dot{1}$; consequently $\frac{2}{99} = .\dot{0}\dot{2}$, $\frac{3}{99} = .\dot{0}\dot{3}$, &c.; and $\frac{2}{999} = .\dot{0}0\dot{2}$, $\frac{3}{999} = .\dot{0}0\dot{3}$, &c. and the same will hold universally.

† In like manner for a mixed circulate; consider it as divisible into its finite and circulating parts, and the same principle will be seen to run through them also; thus, the mixed circulate $.1\dot{6}$ is divisible into the finite decimal .1, and the repetend $.0\dot{6}$ but $.1 = \frac{1}{10}$ and $.0\dot{6}$ would be

2. Multiply the nines in the said denominator by the finite part, and add the repeating decimal to the product for the numerator.

3. If the repetend begins in some integral place, the finite value of the circulating part must be added to the integral part.

EXAMPLES.

1. What is the vulgar fraction equivalent to $.13\dot{8}$.

$9 \times 13 + 8 = 125 =$ *numerator*, and 900 *the denominator* $\therefore .13\dot{8} = \frac{125}{900} = \frac{5}{36}$ *the answer.*

2. What is the least vulgar fraction equivalent to $.5\dot{3}$?

Ans. $\frac{8}{15}$

3. What is the least vulgar fraction equal to $.5\dot{9}2\dot{5}$?

Ans. $\frac{16}{27}$

4. What is the least vulgar fraction equal to $.008\dot{4}9713\dot{3}$?

Ans. $\frac{83}{9768}$

CASE 3.

To make any number of dissimilar repetends similar and conterminous.

RULE*.

Change them into other repetends, which shall each consist of as many figures as the least common multiple of the several numbers of places, found in all the repetends, contains units.

---

be $= \frac{6}{9}$ provided the circulation began immediately after the place of units; but as it begins after the place of tens, it is $\frac{6}{9}$ of $\frac{1}{10} = \frac{6}{90}$, and so the vulgar fraction $= .1\dot{6}$ is $\frac{1}{10} + \frac{6}{90} = \frac{9}{90} + \frac{6}{90} = \frac{15}{90}$, and is the same as by the rule.

* Any given repetend whatever, whether single, compound, pure, or mixed, may be transformed into another repetend, that shall consist of an equal, or greater number of figures at pleasure: thus $.\dot{4}$ may be transformed to $.\dot{4}\dot{4}$, or $.\dot{4}4\dot{4}$, or $.\dot{4}\dot{4}$, &c. Also $.\dot{5}\dot{7} = .\dot{5}75\dot{7} = .5\dot{7}5\dot{7} = .\dot{5}7\dot{5}$; and so on; which is too evident to need any farther demonstration,

And any circulate may be transformed into another, whose repetend shall begin at any distance after the finite part: thus $.\dot{0}04\dot{6} = .0\dot{0}46\dot{0} = .00\dot{4}60\dot{0} = .004\dot{6}00\dot{4}$.

EXAMPLES.

Dissimilar.		*Made similar and conterminous.*
$9.\dot{8}1\dot{4}$	=	$9.81\dot{4}8148\dot{1}$
1.5	=	$1.5\dot{0}00000\dot{0}$
$87.2\dot{6}$	=	$87.2\dot{6}66666\dot{6}$
$.08\dot{3}$	=	$.08\dot{3}3333\dot{3}$
$124.\dot{0}\dot{9}$	=	$124.09\dot{0}909\dot{0}9$

2. Make $.\dot{3}$ $.\dot{2}7$ and $.0\dot{4}5$ similar and conterminous.
3. Make $.\dot{3}2\dot{1}$, $.8\dot{2}6\dot{2}$; $.\dot{0}\dot{5}$ and $.0\dot{9}0\dot{2}$ similar and conterminous.
4. Make $.5\dot{2}17$, $3.\dot{6}4\dot{3}$ and $17.\dot{1}2\dot{3}$ similar and conterminous.

CASE 4.

To find whether the decimal fraction, equal to a given vulgar one, be finite or infinite, and how many places the repetend will consist of.

RULE*.

1. Reduce the given fraction to its least terms, and divide the denominator by 2, 5 or 10, as often as possible.

2. Divide 9999, &c. by the former result till nothing remains, and the number of 9's used will shew the number of places in the repetend; which will begin after as many places of figures as there were 10's, 2's or 5's divided by.

If the whole denominator vanishes in dividing by 2, 5 or 10, the decimal will be finite, and will consist of as many places as you perform divisions.

EXAM-

* In dividing 1.0000, &c. by any prime number whatever, except 2 or 5, the figures in the quotient will begin to repeat over again as soon as the remainder is 1. And since 9999, &c. is less than 10000, &c. by 1, therefore 9999, &c. divided by any number whatever, will leave 0 for a remainder, when the repeating figures are at that period. Now whatever number of repeating figures we have when the dividend is 1, there will be exactly the same number when the dividend is any other number whatever. For the product of any circulating number, by any other given number, will consist of the same number of repeating figures as before. Thus, let .507650765076, &c. be a circulate whose repeating parts is 5076. Now every repetend (5076) being equally multiplied, must produce the same product. For though these products will consist of

EXAMPLES.

1. Required to find whether the decimal equal to $\frac{210}{1120}$ be finite or infinite, and if infinite, how many places that repetend will consist of.

$$\textit{First}\ 10)\frac{210}{1120} = \frac{21}{112}, 2)\ 112 = \overset{(2)}{56} = \overset{(2)}{28} = \overset{(2)}{14} = 7$$

Then $7\overline{)999999}$ 142857; *and therefore the decimal is infinite, and the circulate consists of 6 places, beginning at the decimal point.*

2. Let $\frac{1}{11}$ be the fraction proposed.
3. Let $\frac{2}{7}$ be the fraction proposed.
4. Let $\frac{13}{404}$ be the fraction proposed.
5. Let $\frac{1}{8544}$ be the fraction proposed.

ADDITION OF CIRCULATING DECIMALS.

RULE*.

1. Make the repetends similar and conterminous, and find their sum as in common addition.

2. Divide this sum by as many nines as there are places in the repetend, and the remainder is the repetend of the sum; which must be set under the figures added, with cyphers on the left hand, when it has not so many places as the repetends.

3. Carry the quotient of this division to the next column, and proceed with the rest as in finite decimals.

EXAMPLES.

1. Let $3.\dot{6} + 78.3\dot{4}7\dot{6} + 735.\dot{3} + 375 + .\dot{2}\dot{7} + 187.\dot{4}$ be added together.

---

of more places, yet the overplus in each, being alike, will be carried to the next, by which means each product will be easily increased, and consequently every four places will continue alike. And the same will hold for any other number whatever.

Now from hence it appears, that the dividend may be altered at pleasure, and the number of places in the repetend will still be the same; thus, $\frac{1}{11} = \dot{9}\dot{0}$, and $\frac{3}{11}$ or $\frac{1}{11} \times 3 = .\dot{2}\dot{7}$ where the number of places in each are alike, and the same will be true in all cases.

* These rules are both evident from what has been said in reduction.

Dissimilar

Dissimilar		*Sim. and Conterminous.*
$3.\dot{6}$	=	$3.6\dot{6}6666\dot{6}$
$78.\dot{3}47\dot{6}$	=	$78.3\dot{4}7647\dot{6}$
$735.\dot{3}$	=	$735.3\dot{3}3333\dot{3}$
$375.$	=	$375.0\dot{0}0000\dot{0}$
$.\dot{2}\dot{7}$	=	$0.2\dot{7}2727\dot{2}$
$187.\dot{4}$	=	$187.4\dot{4}4444\dot{4}$
		$1380.0\dot{6}4819\dot{3}$ *the Product.*

In this question the sum of the repetends is 2648191, which divided by 999999 gives 2, to carry, and the remainder is 0648193.

2. Let $5391.357 + 72.3\dot{8} + 187.2\dot{1} + 4.29\dot{6}5 + 217.849\dot{6} + 42.17\dot{6} + .52\dot{3} + 58.3\dot{0}04\dot{8}$ be added together.

Ans. $5974.1037\dot{1}$

3. Add $9.\dot{8}1\dot{4} + 1.5 + 87.2\dot{6} + 0.8\dot{3} + 124.\dot{0}\dot{9}$ together.

Ans. $222.7\dot{5}57239\dot{0}$

4. Add $\dot{1}6\dot{2} + 134.\dot{0}\dot{9} + 2.\dot{9}\dot{3} + 97.2\dot{6} + 3.\dot{7}6923\dot{0} + 99.\dot{0}8\dot{3} + 1.5 + .\dot{8}1\dot{4}$ together. *Ans.* $501.62\dot{6}5107\dot{7}$

SUBTRACTION OF CIRCULATING DECIMALS.

RULE.

Make the repetends similar and conterminous, and subtract as usual; observing, that if the repetend of the number to be subtracted, be greater than the repetend of the number it is to be taken from, the right-hand figure of the remainder must be made less by unity than it would be if the expressions were finite.

EXAMPLES.

1. From $85.\dot{6}\dot{2}$ take $13.7\dot{6}43\dot{2}$.

$$85.\dot{6}\dot{2} = 85.6\dot{2}62\dot{6}$$
$$13.7\dot{6}43\dot{2} = 13.7\dot{6}43\dot{2}$$

$71.8\dot{6}19\dot{3}$ *the difference.*

2. From $476.\dot{3}\dot{2}$ take $84.7\dot{6}9\dot{7}$. *Ans.* $391.55\dot{2}\dot{4}$
3. From $3.8\dot{5}6\dot{4}$ take $.03\dot{8}\dot{2}$. *Ans.* $3.\dot{8}\dot{1}$

Multiplication of Circulating Decimals.

Rule.

1. Turn both the terms into their equivalent vulgar fractions, and find the product of those fractions as usual.

2. Reduce the vulgar fraction, expressing the product, to an equivalent decimal one, and it will be the product required.

Examples.

Multiply $.\dot{3}\dot{6}$ by $.2\dot{5}$

$$.\dot{3}\dot{6} = \frac{36}{99} = \frac{4}{11}$$

$$.2\dot{5} = \frac{23}{90}$$

$$\frac{4}{11} \times \frac{23}{90} = \frac{92}{990} = .0\dot{9}2\dot{9} \textit{ the product.}$$

2. Multiply $37.2\dot{3}$ by $2\dot{6}$. *Ans.* $9.92\dot{8}$
3. Multiply $8574.\dot{3}$ by $8\dot{7}.5$. *Ans.* $750730.\dot{5}1\dot{8}$
4. Multiply $3.\dot{9}7\dot{3}$ by 8. *Ans.* $31.\dot{7}9\dot{1}$
5. Multiply 49640.54 by $.\dot{7}050\dot{3}$. *Ans.* $34998.4\dot{1}9900\dot{3}$
6. Multiply $3.\dot{1}4\dot{5}$ by $4.\dot{2}9\dot{7}$. *Ans.* $13.5\dot{1}69533$

Division of Circulating Decimals.

Rule.

1. Change both the divisor and dividend into their equivalent vulgar fractions, and find their quotient as usual.

2. Reduce the vulgar fraction, expressing the quotient, to its equivalent decimal, and it will be the quotient required.

Examples.

1. Divide $.\dot{3}\dot{6}$ by $.2\dot{5}$

$$.\dot{3}\dot{6} = \frac{36}{99} = \frac{4}{11}$$

$$.2\dot{5} = \frac{23}{90}$$

$$\frac{4}{11} \div \frac{23}{90} = \frac{4}{11} \times \frac{90}{23} = \frac{360}{253} = 1\tfrac{107}{253} \textit{ the quotient.}$$

2. Divide

2. Divide $319.28007\dot{1}\dot{1}2$ by 764.5. *Ans.* $4176\dot{3}2\dot{5}$
3. Divide $234.\dot{6}$ by $.\dot{7}$. *Ans.* $301.\dot{7}1428\dot{5}$
4. Divide $13.\dot{5}16953\dot{3}$ by $4.\dot{2}9\dot{7}$. *Ans.* $3.\dot{1}4\dot{5}$

DUODECIMALS.

DUODECIMALS, or *Cross Multiplication*, is a rule made use of by workmen and artificers in casting up the contents of their works.

Dimensions are generally taken in feet, inches and parts.

Inches and parts are sometimes called primes, seconds, thirds, &c. and are marked thus: primes ($'$), seconds ($''$), thirds ($'''$), fourths (^{iv}), &c.

Artificers work is computed by different measures, *viz.*

1. Glazing, and mason's flatwork by the foot.
2. Painting, paving, plaistering, &c. by the yard.
3. Partitioning, flooring, roofing, tiling, &c. by the square of 100 feet.
4. Brickwork, &c. by the rod of $16\frac{1}{2}$ feet, whose square is $272\frac{1}{4}$.

Note. Bricklayers always value their work at the rate of a brick and a half thick; and if the wall is more or less, it must be reduced to that thickness.

RULE.

1. Under the multiplicand, write the corresponding denominations of the multiplier.
2. Multiply each term in the multiplicand, beginning at the lowest, by the feet in the multiplier, and write the result of each under its respective term, observing to carry an unit for every 12, from each lower denomination to its next superior.
3. In the same manner, multiply all the multiplicand by the primes in the multiplier, and set the result of each term one place removed to the right-hand of those in the multiplicand.
4. Do the same with the seconds in the multiplier, setting the result of each term two places removed to the right-hand of those in the multiplicand.
5. Proceed in like manner with all the rest of the denominations, and their sum will give the answer required.

EXAMPLES.

1. Multiply 10 *fe.* : 4′ : 5″ by 7 *fe.* : 8′ : 6″

$$
\begin{array}{rcccccccc}
10\,fe. & : & 4' & : & 5'' & & & & \\
7 & : & 8 & : & 6 & & & & \\
\hline
72 & : & 6 & : & 11 & & & & \\
6 & : & 10 & : & 11 & : & 4 & & \\
 & & 5 & : & 2 & : & 2 & : & 6 \\
\hline
79\,fe. & : & 11' & : & 0'' & : & 6''' & : & 6^{iv}
\end{array}
$$
anſwer.

2. Multiply 4 *fe.* : 6′ by 14 *fe.* : 9′. *Anſ.* 66 *fe.* : 4′ : 6″.
3. What is the product of 39 *fe.* : 10′ : 7″ by 18 *fe.* 8′ : 4″? *Anſ.* 745 *fe.* : 6′ : 10″ : 2‴ : 4^{iv}.
4. Multiply 24 *fe.* : 10′ : 8″ : 7‴ : 5^{iv} by 9 *fe.* : 4′ : 6″. *Anſ.* 233 *fe.* : 4′ : 5″ : 9‴ : 6^{iv} : 4^{v} : 6^{vi}.
5. Multiply 368 *fe.* : 7′ : 5″ by 137 *fe.* : 8′ : 4″. *Anſ.* 50756 *fe.* : 7′ : 10″ : 9‴ : 8^{iv}.
6. What is the price of a marble ſlab, whoſe length is 5 *fe.* : 7′ and breadth 1 *fo.* : 10′, at 6 *s. per* foot? *Anſ.* 3 *l.* 1 *s.* 5 *d.*
7. There is a houſe with 3 tier of windows, 3 in a tier, the height of the firſt tier is 7 *fe.* : 10′, of the ſecond 6 *fe.* : 8′, and of the third 5 *fe.* 4′, and the breadth of each is 3 *fe.* : 11′: what will the glazing come to at 14 *d. per* foot? *Anſ.* 13 *l.* 11 *s.* 10½ *d.*
8. A room is to be ceiled, whoſe length is 74 *fe.* : 9′, and width 11 *fe.* : 6′: what will it come to at 3 *s.* 10½ *d. per* yard? *Anſ.* 18 *l.* 10 *s.* 1 *d.*
9. What will the paving a court yard come to at 4¾ *d. per* yard, the length being 58 *fe.* : 6′, and breadth 54 *fe.* 9′? *Anſ.* 7 *l.* 0 *s.* 10 *d.*
10. A room is 97 *fe.* : 8′ about, and 9 *fe.* : 10′ high: what will the painting of it come to, at 2 *s.* 8¾ *d. per* yard? *Anſ.* 14 *l.* 11 *s.* 2½ *d.*
11. A piece of wainſcotting is 8 *fe.* : 3′ long, and 6 *fe.* : 6′ broad: what will it come to at 6 *s.* 7½ *d. per* yard? *Anſ.* 1 *l.* 19 *s.* 5 *d.*
12. If a houſe meaſures within the walls 52 *fe.* : 8′ in length, and 39 *fe.* : 6′ in breadth, and the roof be of a true pitch, or the rafters ¾ of the breadth of the building, what will it come to roofing at 10 *s.* 6 *d. per* ſquare? *Anſ.* 12 *l.* 12 *s.* 11¾ *d.*

13. What

13. What will the tyling of a barn coſt at 25*s.* 6*d.* *per* ſquare, the length being 43 *fe.* 10′ and the breadth 27 *fe.* : 5′ on the flat, the eave boards projecting 16 inches on each ſide?

Anſ. 24*l.* 9*s.* $5\frac{1}{2}$*d.*

14. How many ſquare rods are there in a wall $62\frac{1}{2}$ feet long, 14 *fe.* : 8′ high, and $2\frac{1}{2}$ bricks thick? *Anſ.* 5 *rods* 167 *fe.*

15. If a garden wall be 254 feet round, and 12 *fe.* : 7′ high, and 3 bricks thick, how many rods does it contain?

Anſ. 23 *rods*, 136 *fe.*

SIMPLE INTEREST BY DECIMALS.

RULE*.

Multiply the principal, ratio, and time together, and it will give the intereſt required.

* The following theorems will ſhew all the poſſible caſes of ſimple intereſt, where $p =$ principal, $t =$ time, $r =$ ratio, and $a =$ amount,

1. $ptr + p = a.$

2. $\dfrac{a}{tr + 1} = p.$

3. $\dfrac{a - p}{rp} = t.$

4. $\dfrac{a - p}{tp} = r.$

A TABLE *ſhewing the number of days from any day of one month to the ſame day of any other month.*

	From any day of											
To	Jan.	Feb.	Mar	Apr	May	June	July	Aug	Sept	Oct.	Nov	Dec.
Jan.	365	334	306	275	245	214	184	153	122	92	61	31
Feb.	31	365	337	306	276	245	215	184	153	123	92	62
Mar	59	28	365	334	304	273	243	212	181	151	120	90
Apr	90	59	31	365	335	304	274	243	212	182	151	121
May	120	89	61	30	365	334	304	273	242	212	181	151
June	151	120	92	61	31	365	335	304	273	243	212	182
July	181	150	122	91	61	30	365	334	303	273	242	212
Aug	212	81	153	122	92	61	31	365	334	304	273	243
Sept	243	212	184	153	123	92	62	31	365	335	304	274
Oct.	273	242	214	183	153	122	92	61	30	365	334	304
Nov	304	273	245	214	184	153	123	92	61	31	365	335
Dec	334	303	275	244	214	183	153	122	91	61	30	365

Note. In leap-year, if the end of the month of February be in the time, one day muſt be added on that account.

RATIO is the simple interest of 1*l.* for 1 year, at the rate *per cent.* agreed on; thus the ratio

$$\text{at}\begin{cases}3 & \text{——} & .03\\ 3\frac{1}{2} & \text{——} & .035\\ 4 & \textit{per cent. is} & .04\\ 4\frac{1}{2} & \text{——} & .045\\ 5 & \text{——} & .05\end{cases}$$

EXAMPLES.

1. What is the interest of 945*l.* 10*s.* for 3 years, at 5 *per cent. per annum?*

$$\begin{array}{r} 945.5 \\ .05 \\ \hline 47.275 \\ 3 \\ \hline 141.825 \\ 20 \\ \hline 16.500 \\ 12 \\ \hline 6.000 \end{array}$$

Anf. 141 *l.* 16 *s.* 6 *d.*

2. What is the interest of 796*l.* 15*s.* for 5 years, at 4 *per cent. per ann.?* *Anf.* 179*l.* 5*s.* $4\frac{1}{2}$*d.*
3. What is the simple interest of 880*l.* for $1\frac{1}{4}$ years, at $3\frac{1}{2}$ *per cent. per ann.?* *Anf.* 38*l.* 10*s.*
4. What is the interest of 537*l.* 15*s.* from November 11th, 1764, to June 5th, 1765, at $3\frac{5}{8}$ *per cent.?* *Anf.* 11 *l.* 0*s.* $\frac{1}{4}$*d.*

DISCOUNT BY DECIMALS.

RULE*.

As the amount of 1*l.* for the given time, is to 1*l.* fo is the intereft of the debt for the faid time, to the difcount required.

Subtract

* Let *s* reprefent any fum or debt, and *t* the time of payment; then will the following table exhibit all the variety that can happen with refpect to prefent worth and difcount.

The

Subtract the discount from the principal, and the remainder will be the present worth.

EXAMPLES.

1. What is the discount of 573 *l.* 15 *s.* due 3 years hence, at $4\frac{1}{2}$ *per cent. per annum.*

$.045 \times 3 + 1 = 1.135 =$ *amount of* 1 *l. for the given time.*
And $573.75 \times .045 \times 3 = 77.45625 =$ *interest of the debt for the given time.*

$$1.135 \quad : \quad 1 \quad :: \quad 77.45625$$

$$
\begin{array}{r}
1.135)77.45625(68.243 \\
6810 \\
\hline
9356 \\
9080 \\
\hline
2762 \\
2270 \\
\hline
4925 \\
4540 \\
\hline
3850 \\
3405 \\
\hline
445
\end{array}
$$

$68.243 = 68$ *l.* 4 *s.* $10\frac{1}{4}$ *d. the answer.*

2. What

The present worth of any sum *s*, at simple interest.			
Rate *per cent.*	For *t* years.	*t* months.	*t* days.
r per cent.	$\frac{100s}{tr + 100}$	$\frac{1200s}{tr + 1200}$	$\frac{36500s}{tr + 36500}$

The discount of any sum *s*, paid before it is due,			
Rate *per cent.*	For *t* years.	*t* months.	*t* days.
r per cent.	$\frac{str}{tr + 100}$	$\frac{str}{tr + 1200}$	$\frac{str}{tr + 36500}$

Of

2. What is the discount of 725*l.* 16*s.* for 5 months at $3\frac{7}{8}$ *per cent. per annum?* *Ans.* 11*l.* 10*s.* $3\frac{1}{4}$*d.*

3. What ready money will discharge a debt of 1377*l.* 13*s.* 4*d.* due 2 years, 3 quarters and 25 days hence, discounting at $4\frac{3}{8}$ *per cent. per annum?* *Ans.* 1226*l.* 8*s.* $8\frac{1}{2}$*d.*

EQUATION OF PAYMENTS BY DECIMALS.

HAVING two debts due at different times, to find the equated time to pay the whole at once.

RULE*.

1. To the sum of both payments, add the continual product of the first payment, the rate, or interest of 1*l.* for 1 year, and the time between the payments, and call this the first number.

2. Divide the first number by twice the product of the first payment and the rate, and call the quotient the second number.

3. Divide

Of the present worth of money paid before it is due at simple interest.

The present worth of any sum m.			
Rate *per cent.*	For n years.	n months.	n days.
r *per cent.*	$\frac{100\,m}{nr+100}$	$\frac{1200\,m}{nr+1200}$	$\frac{36500\,m}{nr+36500}$

Of discounts to be allowed for paying of money before it falls due at simple interest.

The discount of any sum m.			
Rate *per cent.*	For n years.	n months.	n days.
r *per cent.*	$\frac{mnr}{nr+100}$	$\frac{mnr}{nr+1200}$	$\frac{mnr}{nr+36500}$

* No rule in arithmetic has been the occasion of so many disputes as that of Equation of Payments. Almost every writer upon this subject has endeavoured to show the fallacy of the methods made use of by others, and to substitute a new one in their stead. But the only true rule, as it appears to me, is that given by Mr. *Malcolm in page* 621 *of his Arithmetic*, the principles of which are derived from the consideration of interest and discount.

The

3. Divide the product of the second payment and the time between the payments, by the product of the first payment and the rate, and call the quotient the third number.

4. From the square of the second number take the third, and call the square root of the difference the fourth number; then the difference of the second and fourth number will be the equated time, after the first payment is due.

EXAMPLES.

1. One hundred pounds is payable 1 year hence, and 105 *l.* 3 years hence: what is the equated time to pay the whole, allowing simple interest at 5 *per cent. per annum?*

First, $100 + 105 + (100 \times .05 \times 2) = 100 + 105 + (5.00 \times 2) = 100 + 105 + 10 = 215 =$ *1st. number.*

Secondly, $215 \div (100 \times 2 \times .05) = 215 \div (5.00 \times 2) = 215 \div 10 = 21.5 =$ *2d. number.*

The rule, given above, is the same as Mr. *Malcolm*'s, except that it is not incumbered with the time before any payment is due, that being no necessary part of the operation.

Demon. of the Rule. Suppose a sum of money to be due immediately, and another sum at the expiration of a certain given time forward, and that it is proposed to find a time to pay the whole at once, so that neither party shall sustain loss.

Now it is plain, that the equated time must fall between the two payments; and that what is got by keeping the first debt after it is due, should be equal to what is lost by paying the second debt before it is due.

But the gain arising from the keeping of a sum of money after it is due, is, evidently, equal to the interest of the debt for that time; and the loss which is sustained by the paying of a sum of money before it is due, is, evidently, equal to the discount of the debt for that time.

It is therefore obvious, that the debtor must retain the sum immediately due, or the first payment, till its interest shall be equal to the discount of the second sum for the time it is paid before due; because, in that case, the gain and loss will be equal, and consequently neither party can be the loser.

Now, to find such a time, let $a =$ 1st. payment, $b =$ second, and $t =$ time between the payments; $r =$ rate, or interest of 1 *l.* for a year, and $x =$ equated time after the first payment.

Then $arx =$ interest of a for x time; and $(btr - brx) \div (1 + tr - rx)$ $=$ discount of b for the time $t - x$.

But $arx = (btr - brx) \div (1 + tr - rx)$ by the question; from which equation, if n be put $= (a + b) \times \frac{1}{ar}$, and $m = bt \times \frac{1}{ar}$, we shall have $x = \frac{1}{2}(t + n) \pm \frac{1}{2}\sqrt{(t + n)^2 - 4m}$.

And

Thirdly, $105 \times 2 \div 100 \times .05 = 210 \div 5.00 = 42 = 3d.$ *number.*

Fourthly, $\sqrt{21.5^2 - 42} = \sqrt{462.25 - 42} = \sqrt{420.25} = 20.5 = 4th.$ *number.*

Then, $21.5 - 20.5 = 1 =$ *equated time from the 1st. payment; and, therefore, 2 years = whole equated time.*

2. Suppose 400*l.* is to be paid at the end of 2 years, and 2100*l.* at the end of 8 years: what is the equated time for one payment, reckoning 5 *per cent.* simple interest?

Ans. 7 *years.*

3. Suppose 300*l.* is to be paid at 1 year's end, and 300*l.* more at the end of $1\frac{1}{2}$ years; it is required to find the time to pay it at one payment, allowing 5 *per cent.* simple interest. *Ans.* 1.248637 *years.*

4. A hundred pounds is to be paid at the end of $2\frac{1}{2}$ years, and another 100*l.* at the end of $3\frac{1}{2}$ years; required the equated time to pay the whole?

COM-

And, since $\frac{1}{2}(t+n)$, or its equal $\frac{1}{2}\sqrt{(t+n)^2}$, is evidently greater than $\frac{1}{2}\sqrt{(t+n)^2 - 4m}$, it is plain that x will have two affirmative values, the quantities $\frac{1}{2}(t+n) + \frac{1}{2}\sqrt{(t+n)^2 - 4m}$, and $\frac{1}{2}(t+n) - \frac{1}{2}\sqrt{(t+n)^2 - 4m}$ being both positive.

But only one of these values will answer the conditions of the question; and in all cases of this problem x will be $= \frac{1}{2}(t+n) - \frac{1}{2}\sqrt{(t+n)^2 - 4m}$.

For suppose the contrary, and let $x = \frac{1}{2}(t+n) + \frac{1}{2}\sqrt{(t+n)^2 - 4m}$. Then $t - x = t - \frac{1}{2}(t+n) - \frac{1}{2}\sqrt{(t+n)^2 - 4m} = \frac{1}{2}(t-n) - \frac{1}{2}\sqrt{(t+n)^2 - 4m} = \frac{1}{2}\sqrt{(t-n)^2} - \frac{1}{2}\sqrt{(t+n)^2 - 4m} = \frac{1}{2}\sqrt{(t+n)^2 - 4tn} - \frac{1}{2}\sqrt{(t+n)^2 - 4m}$.

But, since $4tn = (at + bt) \times \frac{4}{ar}$, and $4m = bt \times \frac{4}{ar}$, it is evident that $\frac{1}{2}\sqrt{(t+n)^2 - 4m}$ must be greater than $\frac{1}{2}\sqrt{(t+n)^2 - 4tn}$; whence $\frac{1}{2}\sqrt{(t+n)^2 - 4tn} - \frac{1}{2}\sqrt{(t+n)^2 - 4m}$ or its equal $t - x$ will be a negative quantity; and, consequently, x will be greater than t; that is, the equated time will fall beyond the second payment, which is absurd.

From this it appears, that the double sign made use of by Mr. *Malcolm*, and every author since, who has given his method, cannot obtain, and that there is no ambiguity in the problem.

In like manner it might be shewn, that the directions usually given for finding the equated time when there are more than two payments, will not

Compound Interest by Decimals.

Rule*.

1. Find the amount of 1 *l.* for a year at the given rate *per cent.*

2. Involve the amount thus found to such a power as is denoted by the number of years.

3. Multiply this power by the principal, or given sum, and the product will be the amount required.

4. Subtract the principal from the amount, and the remainder will be the interest.

Examples.

1. What is the compound interest of 509*l.* for 4 years at 5 *per cent. per annum.*

1.05

---

not agree with the hypothesis; but this may be easily seen by working an example at large, and examining the truth of the conclusion.

The equated time for any number of payments may be easily found when the question is proposed in numbers; but it would not be easy to give algebraic theorems for those cases, on account of the variation of the debts and times, and the difficulty of finding between which of the payments the equated time would happen.

Supposing r to be the amount of 1 *l.* for 1 year, and the other letters as before, then $t = log.\frac{art + b}{a + b} \div log.r.$ will be a general theorem for the equated time of any two payments, reckoning compound interest, and is found in the same manner as the former.

* *Demon.* Let $r =$ amount of 1 *l.* for 1 year, and $p =$ principal or given sum; then, since r is the amount of 1 *l.* for 1 year, r^2 will be its amount for 2 years, r^3 for 3 years, and so on; for, when the rate and time is the same, all principal sums are necessarily as their amounts; and consequently as r is the principal for the second year, it will be as $1 : r :: r : r^2 =$ amount for the second year, or principal for the third; and again, as $1 : r :: r^2 : r^3 =$ amount for the third year, or principal for the fourth, and so on to any number of years. And if the number of years be denoted by t, the amount of 1 *l.* for t years will be r^t. From hence it will appear, that the amount of any other principal sum p for t years is pr^t; for as $1 : r^t :: p : pr^t$, the same as in the rule.

If the rate of interest be determined to any other time than a year, as $\frac{1}{2}$, $\frac{1}{4}$, &c. the rule is the same, and then t will represent that stated time.

Let $r =$ amount of 1 *l.* for 1 year, at the given rate *per cent.* $p =$ principal, or sum put out to interest; $i =$ interest, $t =$ time, and $m =$ amount for the time t.

Then

$$\begin{array}{r}
1.05 = \textit{amount of 1 l. for 1 year} \\
1.05 \quad \textit{at 5 per cent.} \\
\hline
525 \\
1050 \\
\hline
1.1025 \\
1.1025 \\
\hline
55125 \\
22050 \\
110250 \\
11025 \\
\hline
1.21550625 = \textit{4th. power of } 1.05. \\
500 = \textit{principal.} \\
\hline
607.75312500 = \textit{amount.} \\
500 \\
\hline
107.753125 = 107\,l.\ 15\,s.\ 0\tfrac{3}{4}\,d. = \textit{interest required.}
\end{array}$$

2. What

Then the following theorems will exhibit the solutions of all the cases in compound interest.

1. $pr^t = m$, 2. $pr^t - p = i$,

3. $m \div r^t = p$, 4. $m \div p^t = r$,

But the most convenient way of giving the theorem for the *time*, as well as for all the other cases, will be by logarithms, as follows:

1. $t \times \log. r + \log. p = \log. m$, 2. $\log. m - t \times \log. r = \log. p$.

3. $\dfrac{\log. m - \log. p}{\log. r} = t$, 4. $\dfrac{\log. m - \log. p}{t} = \log. r$.

If the compound interest, or amount of any sum, be required for the parts of a year, it may be determined as follows:

I. When the time is any aliquot part of a year.

RULE.

1. Find the amount of 1 *l.* for 1 year, as before, and that root of it which is denoted by the aliquot part, will be the amount sought.

2. Multiply the amount thus found by the principal, and it will be the amount of the given sum required.

II. When

2. What is the amount of 760*l.* 10*s.* for 4 years, at 4 *per cent?* *Ans.* 889*l.* 13*s.* 6½*d.*

3. What is the compound interest of 760*l.* 10*s.* for 4 years, at 4 *per cent. per annum?* *Ans.* 129*l.* 3*s.* 6[illegible] *d.*

4. What is the amount of 721*l.* for 21 years, at 4 *per cent. per annum?* *Ans.* 1642*l.* 19*s.* 10*d.*

5. What is the amount of 217*l.* forborn 2¼ yeras, at 5 *per cent. per annum*, supposing the interest payable quarterly? *Ans.* 242*l.* 13*s.* 4½*d.*

ANNUITIES.

AN ANNUITY is a sum of money payable every year for a certain number of years, or for ever.

When the debtor keeps the annuity in his own hands, beyond the time of payment, it is said to be in *arrears.*

The sum of all the annuities for the time they have been forborn, together with the interest due upon each, is called the *amount.*

If an annuity is to be bought off, or paid all at once, at the beginning of the first year, the price which ought to be given for it is called the *present worth.*

To find the Amount of an Annuity at Simple Interest.

RULE*.

1. Find the sum of the natural series of numbers 1, 2, 3, &c. to the number of years less one.

2. Multiply

II. When the time is not an aliquot part of a year.

RULE.

1. Reduce the time into days, and the 365th root of the amount of 1*l.* for 1 year, is the amount for 1 day.

2. Raise this amount to that power whose index is equal to the number of days, and it will be the amount for the given time.

3. Multiply this amount by the principal, and it will be the amount of the given sum required.

To avoid extracting very high roots, the same may be done by logarithms, thus: divide the logarithm of the rate, or amount of 1*l.* for 1 year, by the denominator of the given aliquot part, and the quotient will be the logarithm of the root sought.

* *Demon.* Whatever the time is, there is due upon the first year's annuity, as many year's interest as the whole number of years less one; and gradually one less upon every succeeding year to the last but one; upon which there is due only one year's interest, and none upon the last;

O therefore

2. Multiply this sum by one year's interest of the annuity, and the product will be the whole interest due upon the annuity.

3. To this product add the product of the annuity and time, and the sum will be the amount sought.

EXAMPLES.

1. What is the amount of an annuity of 50*l.* for 7 years, allowing simple interest at 5 *per cent.*?

$$1 + 2 + 3 + 4 + 5 + 6 = 21 = 3 \times 7$$

£.	s.	
2	10	= 1 *year's interest of* 50£.
	3	
7	10	
	7	
52	10	
350	0	= 50*l.* × 7

402*l.* 10*s.* = *amount required.*

---

therefore in the whole there is due as many year's interest of the annuity as the sum of the series, 1, 2, 3, 4, &c. to the number of years less one. Consequently one year's interest multiplied by this sum, must be the whole interest due; to which if all the annuities be added, the sum is plainly the amount. *Q. E. D.*

Let r be the ratio, n the annuity, t the time, and a the amount.

Then will the following theorems give the solutions of all the different cases.

1. $\frac{rt^2n - trn}{2} + tn = a.$ 2. $\frac{2a - 2tn}{t^2n - tn} = r,$

3. $\frac{2a}{t^2r - tr + 2t} = n,$ 4. $\sqrt{(\frac{2a}{rn} + \frac{d}{4})} - \frac{d}{2} = t,$

In the last theorem $d = \frac{2n - rn}{rn}$, and in theorem 1st. if a sum cannot be found equal to the amount, the problem is impossible in whole years.

Note, Some writers look upon this method of finding the amount of an annuity as a species of *compound interest*; the annuity itself, they say, being, properly, the simple interest, and the capital, from whence it arises, the principal.

2. If

2. If a pension of 600*l.* *per ann.* be forborn 5 years, what will it amount to, allowing 4 *per cent.* simple interest?

Ans. 3240*l.*

3. What will an annuity of 250*l.* amount to in 7 years, to be paid by half yearly payments, at 6 *per cent* *per annum*, simple interest? *Ans.* 2091*l.* 5*s.*

To find the present Worth of an Annuity at Simple Interest.

RULE*.

Find the present worth of each year by itself, discounting from the time it falls due, and the sum of all these will be the present worth required.

EXAMPLES.

1. What is the present worth of an annuity of 100*l.* to continue 5 years, at 6 *per cent.* *per ann.* simple interest?

106	: 100	:: 100	: 94.3396	=	*present worth for* 1	*year.*
112	: 100	:: 100	: 89.2857	=	. . . . 2*d.*	*year.*
118	: 100	:: 100	: 84.7457	=	. . . . 3*d.*	*year.*
124	: 100	:: 100	: 80.6451	=	. . . . 4*th*	*year.*
130	: 100	:: 100	: 76.9230	=	. . . . 5*th*	*year.*

425.9391 = 425*l.* 18*s.* 9¼*d.* = *present worth of the annuity required.*

2. What

* The reason of this rule is manifest from the nature of discount, for all the annuities may be considered separately, as so many single and independent debts, due after 1, 2, 3, *&c.* years; so that the present worth of each being found, their sum must be the present worth of the whole.

This is *Kersey*'s rule, as it is given in his appendix to *Wingate*'s Arithmetic. Sir *Samuel Moreland*, *Ward*, &c. have represented it as very erroneous, and given another rule, which they say, brings out the true solution.

Now, granting the condition or agreement of allowing simple interest to be consistent, it appears to me that *Kersey*'s rule is the true one, and the error which Sir *Samuel* and others complain of seems to lie all on their side.

But it would be needless to enter further into the merits of this dispute, since the purchasing of annuities by simple interest is in the highest degree unjust and absurd. One instance only will be sufficient to shew the truth of this assertion. The price of an annuity of 50*l.* to continue 40 years, discounting at 5 *per cent.* will, by either of the rules, amount to a sum of which one year's interest only exceeds the annuity. Would it not therefore be highly ridiculous to give, for an annuity to continue

only

2. What is the present worth of an annuity or pension of 500*l.* to continue 4 years, at 5 *per cent. per ann.* simple interest?

Ans. 1782*l.* 5*s.* 7*d.*

To find the Amount of an Annuity at Compound Interest.

RULE*.

1. Make 1 the first term of a geometrical pregression, and the amount of 1*l.* for 1 year, at the given rate *per cent.* the ratio.

2. Carry the series to as many terms as the number of years, and find its sum.

3. Multiply the sum, thus found, by the given annuity, and the product will be the amount sought.

EXAMPLES.

1. What is the amount of an annuity of 40*l.* to continue 5 years, allowing 5 *per cent.* compound interest?

$$1 + 1.05 + 1.05^2 + 1.05^3 + 1.05^4 = 5.52563125$$

$$\begin{array}{r} 5.52563125 \\ 40 \\ \hline 221.025250 \\ 20 \\ \hline 0.505000 \\ 12 \\ \hline 6.060000 \end{array}$$

Ans. 221*l.* 0*s.* 6*d.*

2. If

only 40 years, a sum which would yield a greater yearly interest for ever.

I have here shewn the method of computing annuities by simple interest, merely in compliance to custom; but would have it considered as a matter more of speculation than real use, it being not only customary, but also most equitable to allow compound interest.

Let p = present worth, and the other letters as before.

$$\text{Then} \begin{cases} n \times \left(\frac{1}{1+r} + \frac{1}{1+2r} + \frac{1}{1+3r}, \text{ \&c. to } \frac{1}{1+tr}\right) = p \\ p \div \left(\frac{1}{1+r} + \frac{1}{1+2r} + \frac{1}{1+3r}, \text{ \&c. to } \frac{1}{1+tr}\right) = n. \end{cases}$$

The other two theorems for the time and rate cannot be given in general terms.

* *Demon.* It is plain, that upon the first year's annuity, there will be due as many years compound interest, as the given number of year's less one,

2. If 50*l.* yearly rent, or annuity, be forborn 7 years, what will it amount to at 4 *per cent. per annum*, compound interest?

Ans. 395*l.*

3. If an annuity of 100*l.* be forborn 20 years, what will it amount to, reckoning 5 *per cent.* compound interest?

To find the present Value of Annuities at Compound Interest.

RULE*.

1. Divide the annuity by the ratio, or the amount of 1*l.* for 1 year, and the quotient will be the present worth of 1 year's annuity.

2. Divide the annuity by the square of the ratio, and the quotient will be the present worth of the annuity for 2 years.

3. Find,

one, and gradually one year less upon every succeeding year to that preceding the last, which has but one year's interest, and the last bears no interest.

Let r, therefore, $=$ rate, or amount of 1*l.* for 1 year; then the series of amounts of 1*l.* annuity, for several years, from the first to the last, is $1, r, r^2, r^3$, &c. to r^{t-1}. And the sum of this, according to the rule in geometrical progression, will be $\frac{r^t - 1}{r - 1}$, $=$ amount of 1*l.* annuity for t years. And as all annuities are proportional to their amounts, therefore $1 : (r^t - 1) \div (r - 1) :: n : (r^t - 1) \times \frac{n}{r - 1} =$ amount of any given annuity n. Q. E. D.

Let $r =$ rate, or amount of 1*l.* for 1 year, and the other letters as before, then $(r^t - 1) \times \frac{n}{r - 1} = a$, and $(ar - a) \div (r^t - 1) = n$;

And from these equations all the cases relating to annuities, or pensions in arrears, may be conveniently exhibited in logarithmic terms thus:

1. Log. n + Log. $(r^t - 1)$ — Log. $(r - 1)$ = Log. a,
2. Log. a — Log. $(r^t - 1)$ + Log. $(r - 1)$ = Log. n.
3. Log. $(ar - a + n)$ — Log. n) $\div$ Log. $r = t$.

The expression for the ratio cannot be given in logarithmic terms, but may easily be obtained from any of the rest.

* The reason of this rule is evident from the nature of the question, and what was said upon the same subject in the purchasing of annuities by simple interest.

3. Find, in like manner, the present worth of each year by itself, and the sum of all these will be the value of the annuity sought.

EXAMPLES.

1. What is the present worth of an annuity of 40*l.* to continue 5 years, discounting at 5 *per cent. per annum*, compound interest?

ratio = 1.05)40.00000(38.095 = *present worth for* 1 *year.*
1.05^2 = 1.1025)40.00000(36.281 = *do. for* 2 *yrs.*
1.05^3 = 1.157525)40.00000(34.556 = *do. for* 3 *yrs.*
1.05^4 = 1.215506)40.00000(32.899 = *do. for* 4 *yrs.*
1.05^5 = 1.276218)40.00000(31.342 = *do. for* 5 *yrs.*

173.173 = 173*l.* 3*s.* $5\frac{1}{2}$*d.*
= *whole present worth of the annuity required.*

2. What is the present worth of an annuity of 21*l.* 10*s.* $9\frac{1}{2}$*d.* to continue 7 years, at 6 *per cent. per annum*, compound interest? *Ans.* 120*l.* 5*s.*

3. What is 70*l. per annum*, to continue 59 years, worth in present money, at the rate of 5 *per cent. per annum?* *Ans.* 1321.3021*l.*

To

Let p = present worth of the annuity, and the other letters as before, then

$n + (r^t - 1) \div (r^{t+1} - r^t) = p$, *and* $p \times (r^{t+1} - r) \div (r^t - 1) = n$;

And from these theorems all the cases, where the purchase of annuities is concerned may be exhibited in logarithmic terms, as follows:

1. $\text{Log. } n + \text{Log. } (1 - \frac{1}{r^t}) - \text{Log. } (r - 1) = \text{Log. } p.$

2. $\text{Log. } p + \text{Log. } (r - 1) - \text{Log. } (1 - \frac{1}{r^t}) = \text{Log. } n.$

3. $\text{Log. } n - \text{Log. } (n + p - pr) \div \text{Log. } r = t.$

The same observation may be applied to the logarithm of the ratio as in the last page.

Let t express the number of half years or quarters, n the half year's or quarter's payment, and r the sum of one pound and $\frac{1}{2}$, or $\frac{1}{4}$ year's interest, then all the preceding rules are applicable to half yearly and quarterly payments, the same as to whole years.

The

To find the present Worth of a Freehold Estate, or an Annuity to continue for ever, at Compound Interest.

RULE*.

As the rate *per cent.* is to 100*l.* so is the yearly rent to the value required.

EXAMPLES.

1. An estate brings in yearly 79*l.* 4*s.* what would it sell for, allowing the purchaser $4\frac{1}{2}$ *per cent.* compound interest for his money?

```
4.5  :  100  :  79.2
                 100
              ------
      4.5)7920.0(1760l. the answer.
          45
          ---
          342
          315
          ---
           270
           270
           ---
```

2. What

The amount of an annuity may also be found for years and parts of a year, thus:

1. Find the amount for the whole years as before.
2. Find the interest of that amount for the given parts of a year.
3. Add this interest to the former amount, and it will give the whole amount required.

The present worth of an annuity for years and parts of a year may be found thus:

1. Find the present worth for the whole years as before.
2. Find the present worth of this present worth, discounting for the given parts of a year, and it will be the whole present worth required.

* The reason of this rule is obvious: for since a year's interest of the price which is given for it is the annuity, there can neither more nor less be made of that price than of the annuity, whether it be employed at simple or compound interest.

The same thing may be shewn thus: The present worth of an annuity to continue for ever, is $\frac{n}{r}+\frac{n}{r^2}+\frac{n}{r^3}+\frac{n}{r^4}$, &c. *ad infinitum*, as has been shewn before; but the sum of this series, by the rules of geometrical progression, is $\frac{n}{r-1}$; therefore $r-1 : 1 :: n : \frac{n}{r-1}$ which is the rule.

The

2. What is the price of a perpetual annuity of 40*l.* discounting at 5 *per cent.* compound interest? *Ans.* 800*l.*
3. What is a freehold estate of 75*l.* a year worth, allowing the buyer 6 *per cent.* compound interest for his money? *Ans.* 1250*l.*

To find the present Worth of an Annuity, or Freehold Estate, in Reversion, at Compound Interest.

RULE*.

1. Find the present worth of the annuity as if it were to be entered on immediately.

2. Find the present worth of the last present worth, discounting for the time between the purchase and commencement of the annuity, and it will be the answer required.

EXAMPLES.

1. The reversion of a freehold estate of 79*l.* 4*s.* *per annum*, to commence 7 years hence, is to be sold, what is it worth in ready money, allowing the purchaser $4\frac{1}{2}$ *per cent.* for his money?

```
4.5  :  100  ::  79.2
                  100
                 ------
         4.5)7920.0(1760 = present worth if en-
             45              tered on immediately.
            ----
             342
             315
            ----
              270
              270
             ----
               0
```

and

The following theorems shew all the varieties of this rule.

1. $\dfrac{n}{r-1} = p$. 2. $(r-1) \times p = n$. 3. $\dfrac{n}{p} + 1 = r$.

The price of a freehold estate, or annuity to continue for ever, reckoning simple interest, would be expressed by $\dfrac{1}{1+r} + \dfrac{1}{1+2r} + \dfrac{1}{1+3r} + \dfrac{1}{1+4r}$, *&c. ad infinitum*; but the sum of this series is infinite, or greater than any assignable number, which sufficiently shews the absurdity of using simple interest in these cases.

* This rule is sufficiently true without a demonstration.

Those who wish to be acquainted with the manner of computing the values of annuities upon lives, may consult the writings of Mr. *Demoivre.* Mr.

and $\overline{1.05}|^7$ = 1.360862)1760.000(1293.297 = 1293*l.* 5*s.* 11¼*d.* = *present worth of* 1760*l. for* 7 *years, or the whole present worth required.*

2. Suppose an estate is worth 20*l. per annum*, and a fine of 100*l.* for a lease of 21 years. Now, if the fine be dropped, how much ought the rent to be increased, allowing 5 *per cent.* compound interest? *Ans.* 7*l.* 16*s.*

3. Which is most advantageous, a term of 15 years in an estate of 100*l. per annum*, or the reversion of such an estate for ever, after the expiration of the said 15 years, computing at at the rate of 5 *per cent. per ann.* compound interest? *Ans. The first term of* 15 *years is better than the reversion for ever afterwards by* 75*l.* 18*s.* 7½*d.*

4. Suppose I would add 5 years to a running lease of 15 years to come, the improved rent being 186*l.* 7*s.* 6*d. per ann.*; what ought I to pay down for this favour, discounting at 4 *per cent. per ann.* compound interest?

Ans. 460*l.* 14*s.* 1¾*d.*

ARITHMETICAL PROGRESSION.

Any rank of numbers increasing by a common excess, or decreasing by a common difference, are said to be in *arithmetical progression*; such are the numbers 1, 2, 3, 4, 5, &c. and 7, 5, 3, 1, .8, .6, &c.

The numbers which form the series are called the *terms* of the progression.

Any three of the five following terms being given, the other two may be readily found.

1. The first term, } commonly called the *extremes*.
2. The last term, }
3. The number of terms.
4. The common difference.
5. The sum of all the terms.

Mr. *Simpson*, Dr. *Price*, and Baron *Maseres*, all of whom have handled this subject in a very skilful and masterly manner.

Dr. *Price*'s treatise upon annuities and reversionary payments, and Baron *Maseres*' doctrine of Life Annuities, are excellent performances, and will be found a very valuable acquisition to those whose inclinations lead them to studies of this nature.

PRO-

PROBLEM I.

The first term, the last term, and the number of terms being given, to find the sum of all the terms.

RULE*.

Multiply the sum of the extremes by the number of terms, and half the product will be the answer.

EXAMPLES.

1. The first term of an arithmetical progression is 2, the last term 53, and the number of terms 18; required the sum of the series.

$$\begin{array}{r} 53 \\ 2 \\ \hline 55 \\ 18 \\ \hline 440 \\ 55 \\ \hline 2)990 \\ \hline 495 \end{array}$$

$$\text{Or, } \frac{(53+2)\times 18}{2} = 495 \textit{ the answer.}$$

2. The first term is 1, the last term 21, and the number of terms 11; required the sum of the series. *Ans.* 121
3. How many strokes do the clocks of Venice, which go to 24 o'clock, strike in the compass of a day? *Ans.* 300
4. If 100 stones be placed in a right line, exactly a yard asunder, and the first a yard from a basket, what length of ground will that man go who gathers them up singly, returning with them one by one to the basket? *Ans. 5 miles and 1300 yards.*
5. The first term of an arithmetical series is 1, the last term 1000, and the number of terms 100; what is the sum of the series?

PRO-

* Suppose another series of the same kind with the given one be placed under it in an inverse order; then will the sum of every two corresponding terms be the same as that of the first and last; consequently any one of those sums multiplied by the number of terms, must give the whole

PROBLEM 2.

The first term, the last term, and the number of terms being given, to find the common difference.

RULE*.

Divide the difference of the extremes by the number of terms less 1, and the quotient will be the common difference sought.

EXAMPLES.

1. The extremes are 2 and 53, and the number of terms is 18; required the common difference.

$$\begin{array}{r} 53 \\ 2 \\ \hline 17)51(3 \\ 51 \\ \hline \end{array} \qquad \begin{array}{r} 18 \\ 1 \\ \hline 17 \end{array}$$

Or

$$\frac{53-2}{18-1} = \frac{51}{17} = 3 \text{ the answer.}$$

2. If the extremes be 3 and 19, and the number of terms 9; it is required to find the common difference, and the sum of the whole series. *Ans. The diff. is* 2, *and the sum is* 99.
3. A man is to travel from London to a certain place in 12 days, and to go but 3 miles the first day, increasing every day by an equal excess, so that the last day's journey may be 58 miles; required the daily increase, and the distance of the place from London.

Ans. Daily increase 5, *distance* 366 *miles.*

---

whole sum of the two series, and half that sum will evidently be the sum of the given series: thus,

Let 1. 2. 3. 4. 5. 6. 7. be the given series,
and 7. 6. 5. 4. 3. 2. 1. the same inverted,
then $8+8+8+8+8+8+8=8\times7=56$, and $1+3+4+5+6+7=\frac{56}{2}=28$.

Q. E. I.

* The difference of the first and last terms evidently shows the increase of the first term, by all the subsequent additions, till it becomes equal to the last; and as the number of those additions are one less than the number of terms, and the increase by every addition equal, it is plain that the total increase divided by the number of additions, must give the difference of every one separately; whence the rule is manifest.

PRO-

PROBLEM 3.

Given the first term, the last term, and the common difference to find the number of terms.

RULE*.

Divide the difference of the extremes by the common difference, and the quotient increased by 1 is the number of terms required.

EXAMPLES.

1. The extremes are 2 and 53, and the common difference 3, what is the number of terms?

$$\begin{array}{r} 53 \\ 2 \\ \hline 3)51 \\ \hline 17 \\ 1 \\ \hline 18 \end{array}$$

$$Or, \frac{53-2}{3}+1=18 \textit{ the answer.}$$

---

* By the last problem the difference of the extremes divided by the number of terms less one, gives the common difference; consequently the same divided by the common difference, must give the number of terms less one; hence this quotient, augmented by one, must be the answer to the question.

In any arithmetical progression, the sum of any two of its terms is equal to the sum of any other two terms taken at an equal distance, on contrary sides of the former; or the double of any one term, is equal to the sum of any two terms taken at an equal distance from it on each side.

The sum of any number of terms (n) of the arithmetical series of odd numbers 1, 3, 5, 7, 9, &c. is equal to the square (n^2) of that number.

That is, if 1, 3, 5, 7, 9, &c. be the numbers,

Then will $1^2, 2^2, 3^2, 4^2, 5^2$, &c. be the sums of 1, 2, 3, &c. of those terms;

For, $0+1$, or the sum of 1 term $=1^2$, or 1
$1+3$, or the sum of 2 terms $=2^2$, or 4
$4+5$, or the sum of 3 terms $=3^2$, or 9
$9+7$, or the sum of 4 terms $=4^2$, or 16, &c.

Whence it is plain, that, let n be any number whatever, the sum of n terms will be n^2.

The following table contains a summary of the whole doctrine of arithmetical progression.

GEOMETRICAL PROGRESSION*.

Any ſeries of numbers, the terms of which gradually increaſe or decreaſe by a conſtant multiplication or diviſion, is ſaid to be in *geometrical progreſſion*. Thus, 4, 8, 16, 32, 64, &c. and 81, 27, 9, 3, 1, &c. are ſeries in geometrical progreſſion, the one increaſing by a conſtant multiplication by 2, and the other decreaſing by a conſtant diviſion by 3.

The number by which the ſeries is conſtantly increaſed or diminiſhed, is called the *ratio*.

Any three of the five following terms being given, the reſt may be readily determined.

1. The firſt term, } commonly called the *extremes*.
2. The laſt term, }
3. The number of terms.
4. The ratio.
5. The ſum of all the terms.

PROBLEM I.

Given the firſt term, the laſt term, and the ratio, to find the ſum of the ſeries.

RULE.

---

* Numbers are compared together, to diſcover the relations they have to each other.

There muſt always be two numbers to form a compariſon: the number which is compared, being written firſt, is called the antecedent, and that to which it is compared the conſequent. Thus, if 3 : 6 : : 12 : 24, 3 and 12 are called the antecedents, and 6 and 24 the conſequents. And when the terms of two ratios, making a proportion, ſucceed one another in the manner of a geometrical progreſſion, they are ſaid to be in *continued* geometrical proportion; but when the proportion is broken, or the ratios are taken between ſuch pairs of numbers as do not ſtand together in a geometrical progreſſion, the proportion is ſaid to be *diſcontinued*: Thus, 2 : 4 : : 8 : 16 is in continued proportion, and 2 : 3 : : 10 : 15 in diſcontinued proportion.

Three or four quantities are ſaid to be in *harmonical proportion*, when, in the former caſe, the difference of the firſt and ſecond is to the difference of the ſecond and third as the firſt is to the third; and, in the latter, when the difference of the firſt and ſecond is to the difference of the third and fourth as the firſt is to the fourth. Thus 2, 3 and 6, and 3, 4, 6, 9 are harmonical proportionals.

Four numbers are ſaid to be *reciprocally* or *inverſely proportional*, when the fourth is leſs than the ſecond by as many times as the third is greater than the firſt, or when the firſt is to the third as the fourth to the ſecond, and *vice verſa*. Thus, 2, 9, 6 and 3 are reciprocal proportionals.

RULE*.

Multiply the last term by the ratio, and from the product subtract the first term, and the remainder divided by the ratio less one will give the sum of the series.

EXAMPLES.

1. The first term of a series in geometrical progression is 1, the last term is 2187, and the ratio 3: what is the sum of the series?

$$\begin{array}{r} 2187 \\ 3 \\ \hline 6561 \\ 1 \\ \hline 3-1=2)6560 \\ \hline 3280 \end{array}$$

$$\textit{Or, } \frac{3 \times 2187 - 1}{3-1} = 3280 \textit{ the answer.}$$

2. The

$$\begin{array}{ll} & \text{If } a : b :: c : d \text{ directly.} \\ \text{Then} \left\{ \begin{array}{l} a : c :: b : d \text{ by alternation.} \\ b : a :: d : c \text{ by inversion.} \\ a+b : b :: c+d : d \text{ by composition.} \\ a-b : b :: c-d : d \text{ by division.} \\ a : a+b :: c : c+d \text{ by conversion.} \\ a+b : a-b :: c+d : c-d \text{ mixedly.} \end{array} \right. & \end{array}$$

* In order to demonstrate the truth of the rule, I shall premise the following Lemmas.

LEMMA 1.

In any geometrical progression of three terms, the square of the mean term is equal to the product of the extremes. Thus, in 2, 6, 18, it will be $2 \times 18 = 6^2 = 36$, and the same of any series of three terms.

Demon. It is plain, that in any geometrical series of three terms, the last term will always be equal to the square of the ratio multiplied into the first term; and the second term equal to the first multiplied by the ratio; consequently as the component factors of the product of the extremes are constantly the same as those of the square of the mean, the results of each must be equal. Thus, in the example above, the last term is equal to $3 \times 3 \times 2$, which multiplied by the first is $3 \times 3 \times 2 \times 2 = 36$; and the second term is 3×2, which squared is $3 \times 3 \times 2 \times 2 = 36$. Q. E. D.

Coroll. The middle term is called a geometrical mean between the two extremes, and is always equal to the square root of their product.

LEMMA

2. The extremes of a geometrical progression are 1 and 65536, and the ratio 4: what is the sum of the series?

Ans. 87381

3. The extremes of a geometrical series are 1024 and 59049, and the ratio is $1\frac{1}{2}$: what is the sum of the series?

Ans. 175099

PROBLEM 2.

Given the first term and the ratio, to find any other term assigned.

RULE*.

1. Write down a few of the leading terms of the series, and place their indices over them, beginning with a cypher.

2. Add

LEMMA 2.

In any geometrical series of four terms, the product of the two means is equal to that of the two extremes.—Thus, if 3 : 6 : : 12 : 24, $3 \times 24 = 6 \times 12$.

Demon. It is plain, from the nature of multiplication, that if one factor be increased as many times as the other is diminished, their product will still be the same. Hence, in the above series, as 6 exceeds 3 as many times as 24 exceeds 12, it is manifest, from what was said in the demonstration of the preceding Lemma, that the product of the extremes will aways be equal to that of the means. *Q. E. D.*

Coroll. In any geometrical series consisting of an even number of terms, the product of the means will be equal to the product of the extremes, or any other pair equally distant from them.

And if the series contain an odd number of terms, the square of the mean will be equal to the product of the adjoining extremes, or any two equally distant from them.

Demon. of the rule. Take any series whatever, as 1. 3. 9. 27. 81. 243, &c. multiply this by the ratio, and it will produce the series 3. 27. 81. 243. 729, &c. Now, let the sum of the proposed series be what it will, it is plain, that the sum of the second series will be as many times the former sum as is expressed by the ratio; subtract the first series from the second, and it will give 729—1: which is evidently as many times the sum of the first series as is expressed by the ratio less one; consequently $\frac{729-1}{3-1}$ = sum of the proposed series, and is the rule; or 729 is the last term multiplied by the ratio, 1 is the first term, and 3—1 is the ratio less one; and the same will hold, let the series be what it will. *Q. E. D.*

* *Demon.* In example 1st, where the first term is equal to the ratio, the reason of the rule is evident; for as every term is some power of the ratio, and the indices point out the number of factors, it is plain from the

2. Add together the most convenient indices to make an index less by one than the number expressing the place of the term sought.

3. Multiply the terms of the geometrical series together, belonging to those indices, and make the product a dividend.

4. Raise the first term to a power whose index is one less than the number of terms multiplied, and make the result a divisor.

5. Divide the divided by the divisor, and the quotient will be the term sought.

Note. When the first term of the series is equal to the ratio, the indices must begin with an unit; and, in this case, the product of the different terms, found as before, will give the term required.

EXAMPLES.

1. The first term of a geometrical series is 2, the number of terms 13, and the ratio 2; required the last term.

	1.	2.	3.	4.	5 *indices.*
	2.	4.	8.	16.	32 *leading terms.*

Then $4 + 4 + 3 + 2 =$ *index to* 13*th. term.*
And $16 \times 16 \times 8 \times 4 = 8192$ *the answer.*

In this example the indices must begin with 1, and such of them be chosen as will make up the entire index to the term required.

2. Required the 12th. term of a geometrical series, whose first term is 3, and ratio 2.

0.	1.	2.	3.	4.	5.	6 *indices.*
3.	6.	12.	24.	48.	96.	192 *leading terms.*

Then $6 + 5 =$ *index to* 12*th. term.*
and $192 \times 96 = 18432 =$ *dividend.*

---

the nature of multiplication, that the product of any two terms, will be another term corresponding with the index, which is the sum of the indices standing over those respective terms.

And in the second example, where the series does not begin with the ratio, it appears that every term, after the two first, contains some power of the ratio multiplied into the first term, and therefore the rule, in this case, is equally evident.

The table in page 161 contains all the possible cases of geometrical progression.

Here

Here the number of terms multiplied is 2, *and* $2-1=1$, *is the power to which the term* 3 *is to be raiſed.*

But the 1*ſt power of* 3 *is* 3, *and therefore* $18432 \div 3 = 6144$ *the* 12*th term required.*

3. The firſt term of a geometric ſeries is 1, the ratio 2, and the number of terms 23; required the laſt term.

Anſ. 4194304.

4. A perſon being aſked to diſpoſe of a fine horſe, ſaid he would ſell him on condition of having one farthing for the firſt nail in his ſhoes, 2 farthings for the ſecond, a penny for the third, and ſo on, doubling the price of every nail to 32, the number of nails in his four ſhoes: what would the horſe be ſold for at that rate? *Anſ* 4473924 *l.* 5 *s.* $3\frac{3}{4}$ *d.*

5. One *Seſſa*, an *Indian*, having firſt diſcovered the game of cheſs, ſhewed it to his prince *Shehram*, who was ſo delighted with the invention, that he bid him aſk what he would as a reward for his ingenuity; upon which *Seſſa* requeſted that he might be allowed one grain of wheat for the firſt ſquare on the cheſs-board, two for the ſecond, four for the third, and ſo on, doubling continually, to 64, the whole number of ſquares: now, ſuppoſing a buſhel to contain 640,000 of theſe grains, it is required to find what number of ſhips, each carrying 100 tons burden, might be freighted with the produce.

Anſ. 7205759403, *and about* $\frac{4}{5}$.

Let a = leaſt term, l = greateſt, n = number of terms, s = ſum of all the terms, d = common difference, and r = ratio; then all the various caſes that can happen, both in arithmetical and geometrical progreſſion, may be ſolved by means of the following theorems.

ARITHMETICAL PROGRESSION.	GEOMETRICAL PROGRESSION.
1. $a = l - (n-1).d$	1. $a = l \div r^{n-1}$
2. $d = (l-a) \div (n-1)$	2. $r = (s-a) \div (s-l)$
3. $n = (l-a) \div d + 1$	3. $n = (\text{L}.l - \text{L}.a) \div \text{L}.r + 1$
4. $l = (n-1) \times d + a$	4. $l = a \times r^{n-1}$
5. $s = (a + \overline{n-1} . \frac{1}{2} d) . n$	5. $s = (ar^n - a) \times (r-1)$

If the value of n, in the third caſe of arithmetical progreſſion, be ſubſtituted for n in the fifth caſe, it will give $s = (a + l) \times \frac{1}{2} n$; and if l in the fourth caſe of geometrical progreſſion be ſubſtituted inſtead of its value in the fifth caſe, it will give $s = (rl - a) \div (r-1)$; and the ſame may be done in any other caſe.

INVOLUTION: OR THE RAISING OF POWERS *.

A *power* is the product arising from multiplying any given number into itself continually a certain number of times, thus,

$2 \times 2 = 4$ is the 2d. power, or square of 2.
$2 \times 2 \times 2 = 8$ is the 3d. power, or the cube of 2.
$2 \times 2 \times 2 \times 2 = 16$ is the 4th. power of 2, &c.

The number denoting the power is called the *index*, or the *exponent* of that power.

If two or more powers are multiplied together, their product is that power whose index is the sum of the exponents of the factors: thus,

$2 \times 2 = 4$ the square of 2; $4 \times 4 = 16 = $ 4th. power of 2; and $16 \times 16 = 256 = $ 8th. power of 2, &c.

EXAMPLES.

1. What is the 5th power of 7?

$$
\begin{array}{rl}
7 & \\
7 & \\
\hline
49 & = \textit{2d. power.} \\
7 & \\
\hline
343 & = \textit{3d. power.} \\
7 & \\
\hline
2401 & = \textit{4th. power.} \\
7 & \\
\hline
16807 & = \textit{5th. power.}
\end{array}
$$

* TABLE of the first NINE POWERS of NUMBERS.

1st	2d	3d.	4th.	5th.	6th.	7th.	8th.	9th.
1	1	1	1	1	1	1	1	1
2	4	8	16	32	64	128	256	512
3	9	27	81	243	729	2187	6561	19683
4	16	64	256	1024	4096	16384	65536	262144
5	25	125	625	3125	15625	78125	390625	1953125
6	36	216	1296	7776	46656	279936	1679616	10077696
7	49	343	2401	16807	117649	823543	5764801	40353607
8	64	512	4096	32768	262144	2097152	16777216	134217728
9	81	729	6561	59049	531441	4782969	43046721	387420489

2. What

2. What is the 3d. power of 35? *Anſ.* 42875
3. What is the 4th. power of $\frac{3}{4}$? *Anſ.* $\frac{81}{256}$
4. What is the 5th. power of .029?
Anſ. 00000002051149
5. What is the 6th. power of 6.03?
Anſ. 48073.293078275529
6. What is the 8th. power of $3\frac{2}{5}$?

EVOLUTION: Or the Extracting of Roots *.

The *root* of any number, or power, is ſuch a number, as being multiplied into itſelf a certain number of times, will produce that power. Thus 2 is the ſquare root of 4, becauſe $2 \times 2 = 4$; and 4 is the cube root of 64, becauſe $4 \times 4 \times 4 = 64$; and ſo on.

Any power of a given number may be found exactly, but there are many numbers of which a given root can never be preciſely determined; although, by the help of decimals, we can approximate towards it, to any aſſigned degree of exactneſs.

The roots which approximate are called *ſurd roots*, and thoſe which are perfectly accurate are called *rational roots*: thus the ſquare root of 2 is a ſurd root; and the cube root of 27 is a rational root, being exactly equal to 3.

To Extract the Square Root.

Rule †.

1. Divide the given number into periods of two figures each, by putting a point over the place of units, another over the place of hundreds, and ſo on.

2. Find

---

* Roots are ſometimes denoted by writing the character $\surd$ before the power, with the index of the root againſt it: thus, the third root of 70 is expreſſed $\surd\, 70$, and the ſecond root of it is $\surd\, 70$, the index 2 being always omitted, when the ſquare root is deſigned.

If the power be expreſſed by ſeveral numbers, with the ſign + or — between them, a line is drawn from the top of the ſign over all the parts of it; thus, the third root of 28 — 13 is $\surd\overline{28-13}$.

But all the roots are now generally deſigned like powers, with fractional indices; thus, the ſquare root of 5 is denoted by $5^{\frac{1}{2}}$, the cube root of 19 by $19^{\frac{1}{3}}$, and the fourth root of 40 — 12 by $\overline{40-12}^{\frac{1}{4}}$, &c.

† In order to ſhew the reaſon of the rule, it will be proper to premiſe the following

Lemma. The product of any two numbers can have at moſt but as many places of figures as are in both the factors, and at leaſt but one leſs.

Demon.

2. Find the greateſt ſquare in the firſt period, and ſet its root on the right hand of the given number, after the manner of a quotient figure in diviſion.

3. Subtract the ſquare, thus found, from the ſaid period, and to the remainder annex the following period, for a dividend.

4. Double the root abovementioned for a diviſor; and find how often it is contained in the dividend, excluſive of the place of units; and ſet the reſult both in the quotient and diviſor.

5. Subtract the product of this quotient figure and the diviſor, thus augmented, from the dividend, and to the remainder bring down the next period, for a new dividend.

6. Find a diviſor as before, by doubling the figures already in the root; and from theſe find the next figure of the root, as in the laſt article; and ſo on through all the periods to the laſt.

Note,

Demon. Take two numbers, conſiſting of any number of places, but let them be the leaſt poſſible of thoſe places, *viz.* unity with cyphers, as 1000 and 100; then their product will be 1 with as many cyphers annexed as are in both the numbers, *viz.* 100000; but 100000 has one place leſs than 1000 or 100 together have; and ſince 1000 and 100 were taken the leaſt poſſible, the product of any other two numbers, of the ſame number of places, will be greater than 100000; conſequently the product of any two numbers can have, at leaſt, but one place leſs than both the factors.

Again, take two numbers, of any number of places, that ſhall be the greateſt poſſible of thoſe places, as 999 and 99. Now 999×99 is leſs than 999×100; but 999×100 ($=$ 99900) contains only as many places of figures as are in 999 and 99; therefore 999×99, or the product of any other two numbers conſiſting of the ſame number of places, cannot have more places of figures than are in both its factors.

Coroll. 1. A ſquare number cannot have more places of figures than double the places of the root, and, at leaſt, but one leſs.

Coroll. 2. A cube number cannot have more places of figures than triple the places of the root, and, at leaſt, but two leſs.

The truth of the rule may be ſhewn algebraically, thus:

Let N $=$ number whoſe ſquare root is to be found.

Now, it appears from the lemma, that there will be always as many places of figures in the root as there are points or periods in the given number, and therefore the figures of thoſe places may be repreſented by letters.

Suppoſe N to conſiſt of two periods, and let the figures in the root be repreſented by *a* and *b*.

Then

Note, if there be decimals in the given number, it must be pointed both ways from unity, and the root be made to consist of as many whole numbers and decimals as there are periods belonging to each; and when the figures belonging to the given number are exhausted, the operation may be continued at pleasure by adding cyphers.

It may also be observed, that the best way of doubling the root, is by adding the last figure of it to the last divisor.

Examples.

1. Required the square root of 5499025.

$$
\begin{array}{r|l}
 & \dot{5}4\dot{9}9\dot{0}2\dot{5}(2345 \text{ *the root.*} \\
 & 4 \\
\hline
43 & 149 \\
3 & 129 \\
\hline
464 & 2090 \\
4 & 1856 \\
\hline
4685) & 23425 \\
 & 23425 \\
\hline
\end{array}
$$

Note. When the root is to be extracted to a great number of places, the work may be considerably abbreviated, thus:

Proceed in the extraction after the common method, till you have found one more than half the required number of figures in the root, and, for the rest, divide the last remainder by its corresponding divisor after the manner of the second contraction in division of decimals.

Exam-

Then $\overline{a+b} = a^2 + 2ab + b^2 = N =$ given number; and to find the root of N is the same as finding the root of $a^2 + 2ab + b^2$, the method of doing which is as follows:

$$
\begin{array}{l}
\text{1st. divisor } a)\ a^2 + 2ab + b^2\ (a + b = \text{root.} \\
\qquad\qquad\qquad a^2 \\
\hline
\text{2d. divisor } 2a + b)\ 2ab + b^2 \\
\qquad\qquad\qquad\qquad 2ab + b^2 \\
\hline
\end{array}
$$

Again, suppose N to consist of 3 periods, and let the figures of the root be represented by a, b and c.

Then

EXAMPLE.

Required the square root of 14876.2357.

```
14876.2357(121.96
1
---------
22  | 48
 2  | 44
---------
241 | 476
  1 | 241
---------
2429| 23523
   9| 21861
---------
24386| 166257
    6| 146316
---------
24392)19941  (8176
       428
       185
        15
         1
```

Ans. 121.968176 *the root required.*

3. What is the square root of 106929? *Ans.* 327
4. What is the square root of 152399025? *Ans.* 12345
5. What is the square root of 119550669121? *Ans.* 345761

Then $\overline{a+b+c}^2 = a^2 + 2ab + b^2 + 2ac + 2bc + c^2$, and the manner of finding a, b and c will be as before, thus:

1st. divisor a) $a^2 + 2ab + b^2 + 2ac + 2bc + c^2$ ($a + b + c =$ root.

$$a^2$$

2d. divisor $2a + b$) $2ab + b^2$
$2ab + b^2$

3d. divisor $2a + 2b + c$) $2ac + 2bc + c^2$
$2ac + 2bc + c^2$

Now, the operation, in each of these cases, exactly agrees with the rule, and the same will be found to be true when N consists of any number of periods whatever.

6. What

6. What is the ſquare root of 368863? *Anſ.* 607.34092, &c.

7. What is the ſquare root of 3.1721812? *Anſ.* 1.78106, &c.

8. What is the ſquare root of .00032754? *Anſ.* .01809

9. What is the ſquare root of $\frac{5}{12}$? *Anſ.* .645497

10. What is the ſquare root of $6\frac{2}{5}$? *Anſ.* .2.5298, &c.

11. What is the ſquare root of 10? *Anſ.* 3.162277, &c.

THE EXTRACTION OF THE CUBE ROOT.

RULE 1*.

1. Separate the given number into periods of three figures each, by putting a point over every third figure from the place of units.

2. Find the greateſt cube in the firſt period, and ſet its root on the right hand of the given number, after the manner of a quotient figure in diviſion.

3. Subtract the cube thus found from the ſaid period, and to the remainder annex the following period; and call this the *reſolvend*.

4. Under the reſolvend, put the triple root, and its triple ſquare, the latter being removed one place to the left, and call their ſum the diviſor.

5. Seek how often the diviſor may be had in the reſolvend, excluſive of the place of units, and ſet the reſult in the quotient.

6. Under the diviſor, put the cube of the laſt quotient figure, the ſquare root of it multiplied by the triple root, and the triple of it by the ſquare of the root, each removed one place to the left, and call their ſum the ſubtrahend.

7. Subtract the ſubtrahend from the reſolvend, and to the remainder bring down the next period for a new reſolvend, with which proceed as before, and ſo on till the whole is finiſhed.

Note.

---

* The reaſon of pointing the given number, as directed in the rule, is obvious from Coroll. 2. to the lemma made uſe of in demonſtrating the ſquare root; and the reſt of the operation will be beſt underſtood from the following analytical proceſs:

Suppoſe N, the given number, to conſiſt of two periods, and let the figures in the root be denoted by a and b.

Then

Note. The ſame rule muſt be obſerved for continuing the operation, and pointing for decimals, as in the ſquare root.

EXAMPLES.

1. Required the cube root of 48228544.

48228544 (364
27

21228 *reſolvend.*

9 *triple of* 3.
27 *triple ſquare of* 3.

279 *diviſor.*

216 *cube of* 6.
324 *ſquare of* 6 × *by the triple of* 3.
162 *triple of* 6 × *by the ſquare of* 3.

19656 *ſubtrahend.*

1572544 *ſecond reſolvend.*

108 *triple of* 36.
3888 *triple ſquare of* 36.

38988 *ſecond diviſor.*

64 *cube of* 4.
1728 *ſquare of* 4 × *by the triple of* 36.
15552 *triple of* 4 × *by the ſquare of* 36.

1572544 *ſecond ſubtrahend.*

* *

Anſ. 364 = *root required.*

2. What is the cube root of 389017? *Anſ.* 73
3. What is the cube root of 1092727? *Anſ.* 103

Then $\overline{a+b}^3 = a^3 + 3a^2b + 3ab^2 + b^3 = N =$ given number, and to find the cube root of N is the ſame as to find the cube root of $a^3 + 3a^2b + 3ab^2 + b^3$; the method of doing which is as follows:

4. What is the cube root of 27054036008? *Ans.* 3002
5. Required the cube root of 122615327232. *Ans.* 4968
6. What is the cube root of 146708.483? *Ans.* 52.74
7. What is the cube root of 171.46776406? *Ans.* 5.555, &c.
8. What is the cube root of .0001357? *Ans.* 05138, &c.
9. What is the cube root of $13\frac{2}{3}$? *Ans.* 2.3908
10. What is the cube root of $\frac{15}{5}\frac{20}{130}$? *Ans.* $\frac{2}{3}$
11. What is the cube root of $\frac{2}{3}$? *Ans.* 873, &c.

RULE 2*.

1. Find, by trials, the nearest rational cube to the given number, and call it the assumed cube.

2. Then, as twice the assumed cube added to the given number, is to twice the given number added to the assumed cube, so is the root of the assumed cube to the root required nearly.

3. And by taking the cube of the root thus found, for the assumed cube, and repeating the operation, the root will be had to a still greater degree of exactness.

EXAM-

$a^3 + 3a^2b + 3ab^2 + b^3$ ($a + b =$ *root.*
a^3

$3a^2b + 3ab^2 + b^3$ *resolvend.*

$3a^2$
$+ 3a$

$3a^2 + 3a$ *divisor.*

$3a^2b$
$+ 3ab^2$
$+ b^3$

$3a^2b + 3ab^2 + b^3$ *subtrahend.*

* *

And in the same manner may the root of a quantity consisting of any number of periods whatever be found.

* The methods usually given for extracting the cube root are so exceedingly tedious and difficult to be remembered, that arithmeticians have long wished for a short easy rule that would be more ready and convenient in practice. Sir *Isaac Newton*, Mr. *Simpson*, Mr. *Emerson*, and several

EXAMPLES.

1. Let it be required to find the cube root of 12484.

Here the nearest rational root is 23, and its cube 12167.

Whence	12167		12484
	2		2
	24334		24968
	12484		12167
	36818	:	37135
			23
			111405
			74270

36818)854105(23.198
11774
729
361
30
1

Ans. 23.198 *the root required, which is true to the last place of decimals.*

2. Let it be required to find the cube root of 2.

Here the nearest rational root is 1, and its cube also 1.

Whence, $1 \times 2 + 2 = 4$, *and* $2 \times 2 + 1 = 5$,

therefore, $4 : 5 :: 1 : \frac{5}{4} = 1.25 =$ *root nearly.*

Again,

---

several other mathematicians of the greatest eminence, have invented approximating rules for this purpose; but no one, that I have yet seen, is so simple in its form, or seems so well adapted for general use as that given above.

That it converges extremely fast may be easily shewn, as follows: Let N = given number, a^3 = assumed cube, and x = correction.

Then

Again, the cube of $\frac{5}{4} = \frac{125}{64}$

Whence $\frac{125 \times 2}{64} + 2 \quad : \quad 2 \times 2 + \frac{125}{64} \quad :: \quad \frac{5}{4}$

Or $\frac{250}{64} + 2 \quad : \quad 4 + \frac{125}{64} \quad :: \quad \frac{5}{4}$

Or $\frac{378}{64} \quad : \quad \frac{381}{64} \quad :: \quad \frac{5}{4} \quad : \quad \frac{381 \times 5 \times 64}{64 \times 4 \times 378} =$

$\frac{381 \times 5}{378 \times 4} = \frac{127 \times 5}{126 \times 4} = \frac{635}{504} = 1.259921 =$ *root, which is true in the last figure.*

2. What is the cube root of 157464? *Ans.* 54
3. What is the cube root of 164566592? *Ans.* 548
4. What is the cube of 673373097125? *Ans.* 8765
5. What is the cube root of 7121.1021698? *Ans.* 19.238, &c.
6. What is the cube root of $\frac{4}{9}$? *Ans.* .763, &c.
7. What is the cube root of .0069761218? *Ans.* .19107, &c.
8. What is the cube root of 117? *Ans.* 4.89097

Then $2a^3 + N : 2N + a^3 :: a : a + x =$ root by the rule; and consequently $(2a^3 + N) \times (a + x) = (2N + a^3) \times a$,
Or $2a^4 + 2a^3x + a^4 + 4a^3x + 6a^2x^2 + 4ax^3 + x^4 = 2aN + a^4$; and by transposing the terms, and dividing by $2a$

$N = a^3 + 3a^2x + 3ax^2 + x^3 + x^3 + \frac{x^4}{2a}$, which by neglecting the terms $x^3 + \frac{x^4}{2a}$, as being very small, becomes $N = a^3 + 3a^2x + 3ax^2 + x^3 =$ to the known cube of $a + x$. *Q. E. I.*

This rule I received from Mr. *Reuben Robbins*, who informed me that he had it from the late Mr. *James Dodson*, at the time he was mathematical master of Christ's Hospital; but since that time I have found it to be exactly the same as Dr. *Halley*'s rational formula, except that it is something more commodiously expressed.

Dr. *Halley*'s irrational formula for the cube-root is $\frac{1}{2} a + \frac{1}{2} \sqrt{\frac{4N - a^3}{3a}}$ which is something more accurate than the former, being erroneous in point of excess as the other is in defect.

To Extract the Roots of Powers in General.

Rule*.

Let N be the given power, or number whose root is to be extracted, n the index of that power, A the assumed power, and r its root.

Then, as the sum of $n+1$ times A, and $n-1$ times N, is to the sum of $n+1$ times N and $n-1$ times A, so is the assumed root r, to the root required, nearly.

That is $(n+1).A+(n-1).N : (n+1).N+(n-1).A :: r$: the true root, nearly.

Or, $(n+1).\frac{1}{2}A+(n-1).\frac{1}{2}N : A \backsim N :: r$ the difference between the true root and the assumed root.

Examples.

1. It is required to find the 5th root of 2.

Assume the root $=1$, and its 5th power will, also, be 1.

Then $N=2$, $A=1$, $n=5$, *and* $r=1$

$$\textit{Whence} \left\{\begin{array}{l}(n+1).A=6\\(n-1).N=\underline{8}\\ \qquad\qquad\quad 14\end{array}\right. \textit{and} \left.\begin{array}{l}(n+1).N=12\\(n-1).A=\underline{\ 4}\\ \qquad\qquad\quad 16\end{array}\right\}$$

Therefore $14 : 16 :: 1 : \frac{16}{14}=\frac{8}{7}=1\frac{1}{7}=1.142857=$ *root, nearly.*

Again,

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* The demonstration of this rule, of which that for the cube root is only a particular case, may be easily derived from the binomial theorem, as follows.

Let $N=$ given number, $n=$ index of the root, $r=$ nearest rational root, and $x=$ remaining part.

Then $N=\overline{r+x}^n=r^n+nr^{n-1}x+n.\frac{n-1}{2}r^{n-2}x^2$, &c.

And $\frac{N-r^n}{nr^{n-1}}=x+\frac{n-1}{2}.\frac{x^2}{r}$, &c. where, on account of the smallness of the quantity $\frac{n-1}{2}.\frac{x^2}{r}$, x may be considered as nearly $=\frac{N-r^n}{nr^{n-1}}$

But

Again, assume the root $= \frac{8}{7}$, *and its 5th power will be* $\frac{32768}{16807}$.

Then, $N = 2$, $A = \frac{32768}{16807}$, $n = 5$, *and* $r = \frac{8}{7}$.

$$\textit{Whence} \left\{ \begin{array}{l} (n+1) \cdot \frac{1}{2} A = \frac{98304}{16807} \\ \quad\quad\quad\quad\quad\quad\quad\quad \textit{and} \\ (n-1) \cdot \frac{1}{2} N = \frac{67228}{16807} \end{array} \right\} N - A = \frac{846}{16807}$$

$$\textit{Therefore } \frac{98304}{16807} + \frac{67228}{16807} : \frac{846}{16807} :: \frac{8}{9}$$

$$Or\ 165532 : 846 :: \frac{8}{7} : \frac{846 \times 8}{165532 \times 7} = \frac{1692}{289681}$$

$= 0.005842$.

Consequently $0.005842 + 1.142857 = 1.148699 =$ root required.

2. What

But $N - r^n$ is also $= n . r^{n-1} x + n . \frac{n-1}{2} r^{n-2} x^2$, &c. $= (n r^{n-1} + n . \frac{n-1}{2} r^{n-2} x) \times x$; whence by substituting the former value of x, in this equation we shall have $N - r^n = (n r^{n-1} + \frac{n-1}{2} . \frac{N - r^n}{r}) . x$ $= (\frac{2nr^n}{2r} + \frac{n-1 . N - nr^n + r^n}{2r}) \times x = (\frac{n+1 . r^n + n-1 . N}{2r}) \times x$; consequently $x = \frac{(N - r^n) \times 2r}{n+1 . r^n + n-1 . N}$, and $r + x = r + \frac{(N - r^n) \times 2r}{n+1 . r^n + n-1 . N} =$ $\frac{n+1 . N + n-1 . r^n}{n+1 . r^n + n-1 . N} \times r$; whence $(n+1) . r^n + (n-1) . N : (n+1) . N + (n-1) . r^n :: r : r + x$, which is the same as the rule.

When the index of the power whose root is to be subtracted is a composite number, the following rule will be serviceable:

Take any two or more indices, whose product is the given index, and extract out of the given number a root answering to one of these indices; and then out of this root extract a root answering to another of the indices, and so on to the last.

Thus

2. What is the 3d root of $\frac{1}{2}$? *Ans.* .7937005
3. What is the 4th root of 2? *Ans.* 1·189207
4. What is the 4th root of 97·41? *Ans.* 3.1415999
5. What is the 6th root of 21035·8? *Ans.* 5·254037
6. What is the 6th root of 2? *Ans.* 1·122462
7. What is the 7th root of 21035·8? *Ans.* 4·145392
8. What is the 7th root of 2? *Ans.* 1·104089
9. What is the 8th root of 21035·8? *Ans.* 3·470323
10. What is the 8th root of 2? *Ans.* 1·090508
11. What is the 9th root of 21035·8? *Ans.* 3.022239
12. What is the 9th root of 2? *Ans.* 1·080059
13. What is the 365th root of 1·05? *Ans.* 1.0013366

POSITION.

POSITION is a method of performing such questions as cannot be resolved by the common direct rules, and is of two kinds, called *single* and *double*.

SINGLE POSITION.

SINGLE POSITION teacheth to resolve those questions whose results are proportional to their suppositions.

RULE*.

1. Take any number, and perform the same operations with it as are described to be performed in the question.

2. Then say, as the result of the operation is to the position, so is the result in the question to the number required.

EXAM-

Thus the fourth root = square root of the square root.

The sixth root = square root of the cube root, &c.

The proof of all roots is by involution, or by casting out the nines as in multiplication.

The following theorems may sometimes be found useful in extracting the root of a vulgar fraction; $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}} = \frac{\sqrt{ab}}{b} = \frac{a}{\sqrt{ab}}$.

* Such questions properly belong to this rule as require the multiplication or division of the number sought by any proposed number; or when it is to be increased or diminished by itself, or any parts of itself, a certain proposed number of times. For in this case the reason of the rule is obvious; it, being, then, evident, that the results are proportional to the suppositions

Thus,

EXAMPLES.

1. A's age is double of B's, and B's is triple of C's, and the sum of all their ages is 140: what is each person's age?

Suppose A's age to be 60

Then will B's $= \frac{60}{2} = 30$

And C's $= \frac{30}{3} = 10$

100 *sum.*

As $100 : 60 :: 140 : \frac{140 \times 60}{100} = 84 =$ *A's age.*

Conseq. $\frac{84}{2} = 42 =$ *B's*

And $\frac{42}{3} = 14 =$ *C's*

140 *Proof.*

2. A certain sum of money is to be divided between 4 persons, in such a manner, that the first shall have $\frac{1}{3}$ of it; the second $\frac{1}{4}$; the third $\frac{1}{6}$; and the fourth the remainder, which is 80*l.* what was the sum? *Ans.* 112*l.*
3. A person after spending $\frac{1}{3}$ and $\frac{1}{4}$ of his money, had 60*l.* left: what had he at first? *Ans.* 144*l.*
4. What number is that which being increased by $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{3}{4}$ of itself, the sum shall be 155? *Ans.* 60
5. A person bought a chaise, horse, and harness, for 60*l.*; the horse came to twice the price of the harness, and the chaise to twice the price of the horse and harness: what did he give for each? *Ans.* 13*l.* 6*s.* 8*d.* *for the horse,* 6*l.* 13*s.* 4*d.* *for the harness, and* 40*l.* *for the chaise.*
6. A vessel has 3 cocks, A, B and C; A can fill it in 1 hour, B in 2, and C in 3: in what time will they all fill it together? *Ans.* $\frac{6}{11}$ *hours.*

$$\text{Thus, } \begin{cases} nx : x :: na : a \\ \frac{x}{n} : x :: \frac{a}{n} : a \\ \frac{x}{n} \pm \frac{x}{m}, \text{\&c.} : x :: \frac{a}{n} \pm \frac{a}{m}, \text{\&c.} : a, \text{ and so on.} \end{cases}$$

Note, 1 may be made a constant supposition in all questions; and in most cases it is better than any other number.

DOUBLE POSITION.

DOUBLE POSITION teacheth to refolve queftions by making two fuppofitions of falfe numbers.

RULE*.

1. Take any two convenient numbers, and proceed with each according to the conditions of the queftion.

2. Find how much the refults are different from the refult in the queftion.

3. Multiply each of the errors by the contrary fuppofition, and find the fum and difference of the products.

4. If the errors are alike, divide the difference of the products by the difference of the errors, and the quotient will be the anfwer.

5. If the errors are unlike, divide the fum of the products by the fum of the errors, and the quotient will be the anfwer.

Note, The errors are faid to be alike, when they are both too great or both too little; and unlike, when one is too great and the other too little.

EXAMPLES.

1. A lady bought tabby at 4*s.* a yard, and perfian at 2*s.* a yard; the whole number of yards fhe bought were 8, and the whole price 20*s*: how many yards had fhe of each fort?

Suppofe 4 yards of tabby, value 16*s.*
Then fhe muft have 4 yards of perfian, value 8

Sum of their values 24
So that the firft error is + 4

Again, fuppofe fhe had 3 yards of tabby at 12*s.*
Then fhe muft have 5 yards of perfian at 10

Sum of the values 22

So that the fecond error is + 2

Then

* The rule is founded on this fuppofition, that the firft error is to the fecond, as the difference between the true and firft fuppofed number, is to the difference between the true and fecond fuppofed number: when that is not the cafe, the exact anfwer to the queftion cannot be found by this rule.

That

Then $4 - 2 = 2 =$ *difference of the errors.*
Also $4 \times 2 = 8 =$ *product of the first supposition and second error.*
And $3 \times 4 = 12 =$ *product of the second supposition by the first error.*
And $12 - 8 = 4 =$ *their difference.*

$$\left.\begin{array}{l}\textit{Whence } 4 \div 2 = 2 = \textit{yards of tabby.}\\ \textit{And } 8 - 2 = 6 = \textit{yards of persian.}\end{array}\right\} \textit{the ans.}$$

2. Two persons, A and B, have both the same income: A saves $\frac{1}{5}$ of his yearly; but B, by spending 50*l.* *per annum* more than A, at the end of 4 years finds himself 100*l.* in debt: what is their income, and what do they spend *per annum?* *Ans. Their income is* 125*l. per ann. also A spends* 100*l. and B* 150*l. per annum.*
3. Two persons, A and B, lay out equal sums of money in trade; A gains 126*l.* and B loses 87*l.* and A's money is now double of B's: what did each lay out? *Ans.* 300*l.*
4. A labourer was hired for 40 days, upon this condition, that he should receive 20*d.* for every day he wrought, and forfeit 10*d.* for every day he was idle: now he received at last 2*l.* 1*s.* 8*d.*: how many days did he work, and how many was he idle? *Ans. wrought* 30 *days, and was idle* 10.
5. A gentleman has two horses of considerable value, and a saddle worth 50*l.*; now, if the saddle be put on the back of the first horse, it will make his value double that of the second; but if it be put on the back of the second, it will make his value triple that of the first: what is the value of each horse? *Ans. One* 30*l. and the other* 40*l.*

6. There

That the rule is true, according to the supposition, may be thus demonstrated.

Let A and B be any two numbers produced from a and b by similar operations; it is required to find the number from which N is produced by a like operation.

Put $x =$ number required, and let $N - A = r$, and $N - B = s$.
Then, according to the supposition on which the rule is founded, $r : s :: x - a : x - b$, whence, by multiplying means and extremes, $rx - rb = sx - sa$; and by transposition $rx - sx = rb - sa$; and by division $x = \frac{rb - sa}{r - s} =$ number sought.

Again,

6. There is a fish whose head is 9 inches long, and his tail is as long as his head and half his body, and his body is as long as his tail and his head: what is the whole length of the fish? *Ans.* 3 *feet*

OF PERMUTATIONS AND COMBINATIONS.

THE COMBINATION OF QUANTITIES, is the shewing how often a less number of things can be taken out of a greater, and combined together, without considering their places, or the order they stand in.

This is sometimes called *election* or *choice*; and here every parcel must be different from all the rest, and no two are to have precisely the same quantities, or things.

The permutation of quantities, is the shewing how many different ways any given number of things may be changed.

This is also called *variation*, *alternation*, or *changes*; and the only thing to be regarded here is the order they stand in; for no two parcels are to have all their quantities placed in the same situation.

The composition of quantities, is the taking a given number of quantities, out of as many equal rows of different quantities, one out of every row, and combining them together.

Here no regard is had to their places; and it differs from combination only, as that admits of but one row of things.

Combinations of the same form, are those in which there are the same number of quantities, and the same repetitions: thus, *abcc*, *bbad*, *deef*, &c. are of the same form; but *abbc*, *abbb*, *aacc*, &c. are of different forms.

PROBLEM I.

To find the number of permutations, or changes, that can be made of any given number of things all different from each other.

Again, if r and s be both negative, we shall have $-r : -s :: x-a : x-b$, and therefore $-rx+rb = -sx+sa$; and $rx-sx = rb-sa$; from whence $x = \frac{rb-sa}{r-s}$ as before.

In like manner, if r or s only be negative, we shall have $x = \frac{rb+sa}{r+s}$, by working as before, which is the rule.

Note, it will be often advantageous to make 1 and 0 the suppositions.

RULE.

R U L E*.

Multiply all the terms of the natural ſeries of numbers, from 1 up to the given number, continually together, and the laſt product will be the anſwer required.

E X A M P L E S.

1. How many changes may be rung on 6 bells?

$$\begin{array}{r} 1 \\ 2 \\ \hline 2 \\ 3 \\ \hline 6 \\ 4 \\ \hline 24 \\ 5 \\ \hline 120 \\ 6 \\ \hline 720 \end{array}$$

Or, $1 \times 2 \times 3 \times 4 \times 5 \times 6 = 720$ *the anſwer*.

2. How many days can 7 perſons be placed in a different poſition at dinner? *Anſ.* 5040 *days*

3. How many changes may be rung on 12 bells, and what time would it require, ſuppoſing 10 changes to be rung in 1 minute, and the year to conſiſt of 365 days, 5 hours and 49 minutes? *Anſ.* 479001600 *changes, and* 91 *years*, 26 *days*, 22 *ho.* 41 *min.*

4. How many changes may be made of the words in the following verſe? *Tot tibi ſunt dotes, virgo, quot ſydera cœlo?* *Anſ.* 40320 *changes.*

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* The reaſon of the rule may be ſhewn thus: any one thing *a* is capable only of one poſition, as *a*.

Any two things *a* and *b*, are only capable of two variations; as *ab*, *ba*; whoſe numbers is expreſſed by 1×2.

If there be 3 things, *a*, *b* and *c*; then any two of them, leaving out the 3d. will have 1×2 variations; and conſequently, when the 3d. is taken in, there will be $1 \times 2 \times 3$ variations.

In the ſame manner, when there are 4 things, every three, leaving out the 4th, will have $1 \times 2 \times 3$ variations; conſequently by taking in ſucceſſively the 4 left out, there will be $1 \times 2 \times 3 \times 4$ variations. And ſo on as far as you pleaſe.

PRO-

PROBLEM 2.

Any number of different things being given; to find how many changes can be made out of them, by taking a given number of quantities at a time.

RULE*.

Take a ſeries of numbers, beginning at the number of things given, and decreaſing by 1 to the number of quantities to be taken at a time, and the product of all the terms will be the anſwer required.

EXAMPLES.

1. How many changes may be rung with 3 bells out of 8?

$$\begin{array}{r} 8 \\ 7 \\ \hline 56 \\ 6 \\ \hline 336 \end{array}$$

Or, $8 \times 7 \times 6$ (= 3 *terms*) = 336 *the anſwer*.

2. How many words can be made with 5 letters of the alphabet, admitting that a number of conſonants alone will not make a word? *Anſ.* 5100480

PRO-

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* This rule expreſſed in terms, is as follows: $m \times (m-1) \times (m-2) \times (m-3)$ &c. to n terms; where m = number of things given, and n = quantities to be taken at a time.

In order to demonſtrate the rule, it will be neceſſary to premiſe the following

LEMMA.

The number of changes of m things, taken n at a time, is equal to m changes of $m-1$ things taken $n-1$ at a time.

Demon. Let any 5 quantities $a\ b\ c\ d\ e$ be given.

Firſt, leave out the a, and let v = number of all the variations of every two, bc, bd, &c. that can be taken out of the 4 remaining quantities $b\ c\ d\ e$.

Now, let a be put in the firſt place of each of them, $a, b, c, a, b, d,$ &c. and the number of changes will ſtill remain the ſame; that is, v = number of variations of every 3 out of the 5, $a\ b\ c\ d\ e$, when a is firſt.

In like manner, if b, c, d, e be ſucceſſively left out, the number of variations of all the two's will alſo = v; and putting b, c, d, e reſpectively

PROBLEM 3.

Any number of things being given; of which there are several given things of one sort, and several of another, &c. To find how many changes can be made out of them all.

RULE*.

1. Take the series $1 \times 2 \times 3 \times 4$, &c. up to the number of things given, and find the product of all the terms.

2. Take the series $1 \times 2 \times 3 \times 4$, &c. up to the number of given things of the first sort, and the series $1 \times 2 \times 3 \times 4$, &c. up to the number of given things of the second sort, &c.

3. Divide

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tively in the first place, to make 3 quantities out of 5, there will still be v variations as before.

But these are all the variations that can happen of 3 things out of 5, when a, b, c, d, e are successively put first; and therefore the sum of all these is the sum of all the changes of 3 things out of 5.

But the sum of these is so many times v as is the number of things; that is $5v$, or mv, $=$ all the changes of 3 things out of 5. And the same way of reasoning may be applied to any numbers whatever.

Demon. of the rule. Let any 7 things $a\ b\ c\ d\ e\ f\ g$ be given, and let 3 be the number of quantities to be taken.

Then $m = 7$ and $n = 3$.

Now, it is evident, that the number of changes that can be made by taking 1 by 1 out of 5 things will be 5, which let $= v$.

Then, by the lemma, when $m = 6$ and $n = 2$, the number of changes will $= mv = 6 \times 5$; which let $= v$ a second time.

Again, by the lemma, when $m = 7$ and $n = 3$, the number of changes $= mv = 7 \times 6 \times 5$; that is $mv = m \times (m-1) \times (m-2)$, continued to 3, or n terms. And the same may be shewn for any other numbers.

* This rule is expressed in terms thus: $\dfrac{1 \times 2 \times 3 \times 4 \times 5, \text{\&c. to } m}{1 \times 2 \times 3, \text{\&c. to } p \times 1 \times 2 \times 3, \text{\&c. to } q}$ &c.; where $m =$ number of things given, $p =$ number of things of the first sort, $q =$ number of things of the second sort, &c.

The demonstration may be shewn as follows:

Any 2 quantities, $a\ b$, both different, admit of 2 changes; but if the quantities are the same, or $a\ b$ becomes $a\ a$, there will be only one alternation; which may be expressed by $\dfrac{1 \times 2}{1 \times 2} = 1$.

Any 3 quantities, $a\ b\ c$, all different from each other, afford 6 variations; but if the quantities be all alike, or $a\ b\ c$ becomes $a\ a\ a$, then the 6 variations will be reduced to 1; which may be expressed by $\dfrac{1 \times 2 \times 3}{1 \times 2 \times 3}$ $= 1$. Again, if two of the quantities only are alike, or $a\ b\ c$ becomes

$a\ a\ c$;

3. Divide the product of all the terms of the first series by the joint product of all the terms of the remaining ones, and the quotient will be the answer required.

EXAMPLES.

1. How many variations can be made of the letters in the word *Bacchanalia*?

$$1 \times 2 \; (= \textit{number of } c\textit{'s}) = 2$$

$$1 \times 2 \times 3 \times 4 \; (= \textit{number of } a\textit{'s}) = 24$$

$$1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10 \times 11$$

$$(= \textit{number of letters in the word}) = 39916800$$

$2 \times 24 = 48)39916800(831600$ *the answer.*

151

76

288

2. How many different numbers can be made of the following figures, 1220005555? *Ans.* 12600

3. How many varieties will take place in the succession of the following musical notes, *fa, fa, fa, sol, sol, la, mi, fa*? *Ans.* 3360

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a a c; then the 6 variations will be reduced to these 3, *a a c, c a a*, and *a c a*; which may be expressed by $\frac{1 \times 2 \times 3}{1 \times 2} = 3$.

Any 4 quantities, *a b c d*, all different from each other, will admit of 24 variations; but if the quantities be the same, or *a b c d* becomes *a a a a*, the number of variations will be reduced to one; which is $= \frac{1 \times 2 \times 3 \times 4}{1 \times 2 \times 3 \times 4} = 1$.

Again, if three of the quantities only be the same, or *a b c d* becomes *a a a b*, the number of variations will be reduced to these 4, *a a a b, a a b a, a b a a*, and *b a a a*; which is $= \frac{1 \times 2 \times 3 \times 4}{1 \times 2 \times 3} = 4$.

And thus it may be shewn that if two of the quantities be alike, or the 4 quantities be *a a b c*, the number of variations will be reduced to twelve; which may be expressed by $\frac{1 \times 2 \times 3 \times 4}{1 \times 2} = 12$.

And by reasoning in the same manner, it will appear that the number of changes which can be made of the quantities *a b b c c c* is equal to 60; which may be expressed by $\frac{1 \times 2 \times 3 \times 4 \times 5 \times 6}{1 \times 2 \times 1 \times 2 \times 3} = 60$; and so for any other quantities whatever.

PRO-

PROBLEM 4.

To find the changes of any given number of things, taking a given number at a time; in which there are several given things of one sort, several of another, &c.

RULE*.

1. Find all the different forms of combination of all the given things, taken as many at a time as in the question.

2. Find the number of changes in any form, and multiply it by the number of combinations in that form.

3. Do the same for every distinct form, and the sum of all the products will give the whole number of changes required.

EXAMPLES.

1. How many alternations, or changes, can be made of every 4 letters out of these 8; *aaabbbcc*?

No. of forms.	*No. of changes.*
a^3b, a^3c, b^3a, b^3c	4
a^2b^2, a^2c^2, b^2c^2	6
a^2bc, b^2ac, c^2ab	12

$$\textit{Therefore}\begin{cases}4 \times 4 = 16\\ 3 \times 6 = 18\\ 3 \times 12 = 36\end{cases}$$

70 = *number of changes required.*

---

* The reason of this rule is plain from what has been shewn before, and the nature of the problem.

A rule for finding the number of forms.

1. Place the things so that the greatest indices may be first, and the rest in order.

2. Begin with the first letter, and join it to the second, third, fourth, &c. to the last.

3. Then take the second letter, and join it to the third, fourth, &c. to the last; and so on till they are entirely exhausted, always remembering to reject such combinations as have occurred before; and this will give the combinations of all the two's.

4. Join the first letter to every one of the two's, and the second, third, &c. as before; and it will give the combinations of all the three's.

5. Proceed in the same manner to get the combinations of all the four's, &c. and you will at last get all the several forms of combination, and the number in each form.

2. How many changes can be made of every 8 letters out of these 10; *aaaabbccde*? *Anf.* 22260

3. How many different numbers can be made out of 1 unit, 2 two's, 3 three's, 4 four's, and 5 five's; taken 5 at a time? *Anf.* 2111

PROBLEM 5.

To find the number of combinations of any given number of things, all different from each other, taken any given number at a time.

RULE*.

1. Take the feries 1, 2, 3, 4, &c. up to the number to be taken at a time, and find the product of all the terms.

2. Take a feries of as many terms, decreafing by 1, from the given number, out of which the election is to be made, and find the product of all the terms.

3. Divide the laft product by the former, and the quotient will be the number fought.

EXAM-

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* This rule, expreffed algebraically, is, $\frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \times \frac{m-3}{4}$ &c. to n terms; where m is the number of given quantities, and n thofe to be taken at time.

Demon. of the rule. 1. Let the number of things to be taken at a time be 2, and the things to be combined $= m$.

Now, when m, or the number of things to be combined, is only two, as a and b, it is evident that there can be but one combination, as ab; but if m be increafed by one, or the letters to be combined be 3, as a, b, c, then it is plain that the number of combinations will be increafed by 2, fince with each of the former letters a and b the new letter c may be joined. In this cafe, therefore, it is evident, that the whole number of combinations will be truly expreffed by $1 + 2$.

Again, if m be increafed by one letter more, or the whole number of letters be four, as a, b, c, d; then it will appear that the whole number of combinations muft be increafed by 3, fince with each of the preceding letters the new letter d may be combined. The combinations, therefore, in this cafe, will be truly expreffed by $1 + 2 + 3$.

And in the fame manner, it may be fhewn, that the whole number of combinations of 2, in 5 things, will be $1 + 2 + 3 + 4$; of 1, in 6 things, $1 + 2 + 3 + 4 + 5$; and of 2, in 7, $1 + 2 + 3 + 4 + 5 + 6$, &c. whence, univerfally, the number of combinations of m things, taken 2 by 2, is $= 1 + 2 + 3 + 4 + 5 + 6$, &c. to $(m-1)$ terms.

But the fum of this feries is $= \frac{m}{1} \times \frac{m-1}{2}$; which is the fame as the rule.

2. Let

Examples.

1. How many combinations can be made of 6 letters out of 10?

$1 \times 2 \times 3 \times 4 \times 5 \times 6$ (= *the number to be taken at a time*) = 720

$10 \times 9 \times 8 \times 7 \times 6 \times 5$ (= *same number from* 10) = 151200

$$\begin{array}{l} 720)151200(210 \textit{ the answer.} \\ \quad\;\; \underline{1440} \\ \qquad 720 \\ \qquad \underline{720} \\ \qquad\quad 0 \end{array}$$

2. How many combinations can be made of 2 letters out of the 24 letters of the alphabet? *Ans.* 276

3. A general, who had often been successful in war, was asked by his king what reward he should confer upon him for his services; the general only desired a farthing for every file, of 10 men in a file, which he could make with a body of 100 men; what is the amount in pounds sterling?

Ans. 18031572350*l.* 9*s.* 2*d.*

Pro-

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2. Let now the number of quantities in each combination be supposed to be three.

Then it is plain, that, when $m = 3$, or the things to be combined are a, b, c, there can be only one combination; but if m be increased by 1, or the things to be combined are 4, as a, b, c, d, then will the number of combinations be increased by 3: since 3 is the number of combinations of 2 in all the preceding letters, a, b, c, and with each two of these the new letter d may be combined.

The number of combinations, therefore, in this case, is $1 + 3$.

Again, if m be increased by one more, or the number of letters be supposed 5; then the former number of combinations will be increased by 6, that is, by all the combinations of 2 in the 4 preceding letters, a, b, c, d; since, as before, with each two of these the new letter e may be combined.

The number of combinations, therefore, in this case, is $1 + 3 + 6$.

Whence, universally, the number of combinations of m things, taken 3 by 3, is $1 + 3 + 6 + 10$. &c. to $m - 2$ terms.

But the sum of this series is $= \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3}$; which is the same as the rule.

And

PROBLEM 6.

To find the number of combinations of any given number of things, by taking any given number at a time; in which there are ſeveral things of one ſort, ſeveral of another, &c.

RULE.

1. Find, by trial, the number of different forms which the things to be taken at a time will admit of, and the number of combinations there are in each.

2. Add all the combinations, thus found, together, and the ſum will be the number required.

EXAMPLES.

1. Let the things propoſed be $a\ a\ a\ b\ b\ c$; it is required to find the number of combinations made of every 3 of theſe quantities.

Forms.	*Combinations.*
a^3	1
a^2b, a^2c, b^2a, b^2c	4
abc	1
	6 = *number*

of combinations required.

2. Let $a\ a\ a\ b\ b\ b\ c\ c$ be propoſed; it is required to find the number of combinations of theſe quantities, taken 4 at a time.
Anſ. 10.

3. How many combinations are there in $a\ a\ a\ a\ b\ b\ c\ c\ d\ e$, taking 8 at a time? *Anſ.* 13.

4. How many combinations are there in $a\ a\ a\ a\ a\ b\ b\ b\ b\ b\ c\ c\ c\ c\ d\ d\ d\ d\ e\ e\ e\ e\ f\ f\ f\ g$, taking 10 at a time? *Anſ.* 2819

PROBLEM 7.

To find the compoſitions of any number, in an equal number of ſets, the things themſelves being all different.

And the ſame thing will hold, let the number of things to be taken at a time be what they will; therefore the number of combinations of m things, taken n at a time, will $= \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \times \frac{m-3}{4}$, &c. to n terms. Q. E. D.

RULE.

RULE*.

Multiply the number of things in every set continually together, and the product will be the answer required.

EXAMPLES.

1. Suppose there are 4 companies, in each of which there are 9 men; it is required to find how many ways 9 men may be chosen, one out of each company.

$$\begin{array}{r} 9 \\ 9 \\ \hline 81 \\ 9 \\ \hline 729 \\ 9 \\ \hline 6561 \end{array}$$

Or, $9 \times 9 \times 9 \times 9 = 6561$ *the answer.*

2. Suppose there are 4 companies; in one of which there are 6 men, in another 8, and in each of the other two 9; what are the choices, by a composition of 4 men, one out of each company? *Ans.* 3888

3. How many changes are there in throwing 5 dice? *Ans.* 7776

EXCHANGE.

Exchange is the method of finding what sum of the money of one country is equal to any given sum of the money of another, according to a certain given course of exchange.

The course of exchange is such a variable sum of the money of one place, as is proposed to be given for a certain constant sum of that of another.

The

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* *Demon.* Suppose there are only two sets; then, it is plain, that, every quantity of the one set being combined with every quantity of the other, will make all the compositions, of two things in these two sets; and the number of these compositions is, evidently, the product of the number of quantities in one set by that in the other.

Again, suppose there are three sets; then the composition of two, in any two of the sets, being combined with every quantity of the third, will make all the compositions of three in the three sets. That is, the compositions of two, in any two of the sets, being multiplied by the num-

The par of exchange is ſuch a quantity of the money of one country, as is intrinſically equal to a certain quantity of the money of another; it being one of theſe that is the conſtant ſum to which the courſe is compared.

The money in the banks of foreign places is finer than that which is current in thoſe places; and the difference between any ſum as it is valued in the one or the other is called the *agio*.

The money made uſe of in exchange is generally imaginary; and in moſt places differs conſiderably from the money in which they keep their accounts. It is alſo to be obſerved, that the money made uſe of in exchange, and the money which is current, is very different, as well as that of banco and current.

All the operations in exchange may be performed by the rule of three and practice.

ENGLAND, WITH HOLLAND, FLANDERS AND GERMANY.

Accounts are kept in theſe places in guilders, ſtivers and pennings; or in pounds, ſhillings and pence, as in England.

The money of Holland and Flanders is diſtinguiſhed by the name of *flemiſh*, and they exchange by the pound ſterling.

8 pennings	make one	grote or penny
2 grotes	make one	ſtiver
6 ſtivers	make one	ſchilling
20 ſtivers	make one	florin or guilder
$2\frac{1}{2}$ florins	make one	rix-dollar
6 florins	make one	pound flemiſh

Exchange from 33*s.* 6*d.* to 36*s.* 6*d.* flem. *per pound* ſterling.
Agio from 3 to 6 *per cent.* for current.

To turn current money into banco, and banco money into current.

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ber of quantities in the remaining ſet, will produce the compoſitions of three in the three ſets, which is, evidently, the continual product of all the three numbers in three ſets. And the ſame manner of reaſoning will hold, let the number of ſets be what it will. *Q. E. D.*

The doctrine of permutations, combinations, *&c.* is of very extenſive uſe in different parts of the mathematics; particularly in the calculation of annuities and chances. The ſubject might have been purſued to a much greater length; but what is here done will be found ſufficient for moſt of the purpoſes to which things of this nature are applicable.

RULE,

RULE.

As 100 with the agio added to it, is to 100, so is any given sum current to its value banco.

And as 100, is to 100 with the agio added to it, so is any given sum banco to its value current.

Note, The exchange is supposed to be made in bank money, and therefore current money must always be turned into banco before the exchange can be made.

EXAMPLES.

1. In 96*l.* 6*s.* 11*d.* sterling, how many florins, &c. exchange at 34*s.* 3*d.* flemish *per pound* sterling?

		96*l.*	6*s.*	11*d.*
10*s.*	*is* $\frac{1}{2}$	48	3	$5\frac{1}{2}$
3*s.* 4*d.*	*is* $\frac{1}{3}$	16	1	$1\frac{3}{4}$
10*d.*	*is* $\frac{1}{4}$	4	0	$3\frac{1}{2}$
1*d.*	*is* $\frac{1}{10}$	-	8	- $\frac{1}{4}$
		164	19	10
				6
		989	19	-

Ans. 989 *flor.* 19 *st.*

2. In 612*l.* 14*s.* 9$\frac{1}{2}$*d.* sterling, how many Dutch rix-dollars, exchange 35*s.* 4*d.* $\frac{7}{8}$ flem. *per l.* sterling?

Ans. 2603 *rix-dol.* 18 *st.* 1 *gr.* 5 *pen.*

3. In 3758 *flor.* 15 *st.* current, agio 5 $\frac{5}{8}$ *per cent.* how many pounds sterling, exchange at 35*s.* 11*d.*? *Ans.* 330*l.* 5*s.* 2$\frac{1}{4}$*d.*

4. In 456*l.* 8*s.* sterling, how many rix-dollars current, agio 4 $\frac{5}{8}$, exchange 36*s.* 1$\frac{1}{2}$*d.*? *Ans.* 2069 *rix-dol.* 2 *flor.* 10 *st.*

5. In 2714 *guil.* 15 *st.* how many pounds sterling; exchange at 35*s.* 6*d.* flemish *per* pound sterling?

Ans. 254*l.* 18*s.* 1$\frac{1}{4}$*d.*

6. In 290*l.* 11*s.* 10*d.* sterling, how many pounds flemish; exchange at 33*s.* 10*d.* flem. *per* pound sterling; and agio at 4$\frac{1}{2}$ *per cent.*? *Ans.* 513*l.* 14*s.* 1$\frac{1}{4}$*d.*

7. In 805*l.* 15*s.* flemish, how many pounds sterling; the agio being 4 *per cent.* and exchange 34*s.* 6*d.* flem. *per* pound sterling? *Ans.* 449*l.* 2*s.* 8$\frac{1}{2}$*d.*

8. The

8. The course of exchange, this day March 29, 1787, between London and Amsterdam is 34*s.* 3*d.* at 2 ½ usance, what ought Amsterdam to give at sight, supposing the interest of money to be 4 *per cent.*? *Ans.* 33*s.* 11¾*d.*

HAMBRO.

They keep their accounts at Hambro in marks and sols lub, and exchange by the pound sterling as in Holland:

2 deniers gros	make one	sol lub
6 sol lubs	make one	sol gros
16 sol lubs	make one	mark
2 marks	make one	drittle, or Hambro dollar
3 marks	make one	rix-dollar
7½ marks	make one	livre gros, or pound flem.

Exchange from 32*s.* to 35*s.* flem. *per l.* sterling.

Agio from 18 to 20 *per cent.* for current, and from 30 to 35 *per cent.* for slight.

EXAMPLES.

1. In 3459 *mar.* 10 *sol l.* banco, how many pounds sterling, exchange 36 *sol. g.* 1 *den. per* pound sterling?

```
                      3459 mar. 10 sol gr.
                        16
                      -----
                      20754
                      3460
                      -----
                      55354
                          2
                      -----
36 sol, 1 d. = 433)110708(255 l.
                      2410
                      1458
                       293
                        20
Ans. 255 l. 13 s. 6¼ d.   ----- &c.
```

2. In 255*l.* 13*s.* 6¼*d.* sterling, how many marks, &c. exchange 36 *sol gros*, 1 *den. per* pound sterling? *Ans.* 3459 *mar.* 10 *sol l.*

3. In 536*l.* sterling how many marks; exchange at 36*s.* 4*d.* flemish *per* pound sterling? *Ans.* 7303 *marks.*

4. In

4. In 127*l.* 3*s.* 4*d.* sterling, how many Hambro marks, exchange at 32 $\frac{1}{3}$ *sol gros per* pound sterling?

Ans. 1541 *mar.* 14 $\frac{1}{3}$ *sol lubs.*

5. In 3065 *rix-doll.* 23 *sol lubs*, how many pounds sterling, exchange at 32 *sol gros*, 8 *den. per* pound sterling?

Ans. 750*l.* 14*s.* 7*d.*

6. In 585 *rix-doll.* 1 *sol gros*, slight money, agio 4 $\frac{9}{16}$ *per cent.* exchange 35 *sol gros*, 8$\frac{1}{2}$ *den.* how many pounds sterling?

Ans. 125*l.* 7*s.* 4*d.*

7. In 934*l.* 1*s.* 2$\frac{3}{4}$ *d.* sterling, how many rix-dollars, &c. current, exchange at 33 *sol gros*, 9$\frac{1}{4}$ *den*, agio 118$\frac{1}{2}$.

Ans. 4672 *rix-doll.* 22 *sol lubs.*

8. In 1075 *marks*, 14 *sol lubs* current, agio 8$\frac{3}{8}$ *per cent.* and 384 *doll.* 2 *sol gros* slight, agio 4$\frac{7}{8}$ *per cent.* exchange 35 *sol gros*, 7 *den.* how many pounds sterling? *Ans.* 129*l.* 6*s.* 6$\frac{1}{4}$*d.*

F R A N C E.

Accounts are kept in France in livres, sols and deniers, and they exchange by the ecu, or crown tournois.

12 deniers	make one	sol
20 sols	make one	livre
3 livres	make one	ecu, or crown tournois
10 livres	make one	pistolé
24 livres	make one	louis d'or, or guinea

Exchange from 30*d.* to 32*d.* sterling *per* ecu.

E X A M P L E S.

1. Reduce 3989 *liv.* 13*s.* 9*d.* into pounds sterling, exchange 31$\frac{1}{4}$*d. per* ecu.

	liv.	*s.*	*d.*
	3)3989	13	9
	1329	17	11
d.			
30 *is* $\frac{1}{8}$	166	4	8$\frac{3}{4}$
1 *is* $\frac{1}{30}$	5	10	9$\frac{1}{4}$
$\frac{1}{4}$ *is* $\frac{1}{4}$	1	7	8$\frac{1}{4}$
	173*l.*	3*s.*	2$\frac{3}{4}$*d.* *the answer.*

2. In 471*l.* 17*s.* 4$\frac{3}{4}$*d.* sterling, how many livres tournois, exchange at 31$\frac{1}{2}$*d.* sterling *per ecu?*

Ans. 10785 *liv.* 11 *sols.* 10 *den.*

3. In

3. In 771 *l.* 17 *s.* 6 *d.* sterling, how many French pistoles, exchange $30\frac{7}{8}$ *d. per ecu?* *Ans.* 180

4. What comes 32 *liv.* 13 *s.* 11 *d.* to in London, at $57\frac{1}{2}$ *d. per* crown at Bourdeaux? *Ans.* 58 *l.* 10 *s.* $3\frac{1}{4}$ *d.*

SPAIN.

Accounts are kept in Spain in piastres, rials and marvadies, and they exchange by the piastre or piso.

4 marvadies vellon, or $2\frac{1}{8}$ marvadies of plate	make one	quarta
$2\frac{1}{2}$ quartas, or 34 marvadies vellon	make one	rial vellon
16 quartas, or 34 marvadies of plate	make one	rial of plate (or dollar)
8 rials of plate	make one	piso, piastre, or piece of 8
5 piastres	make one	Spanish pistole
2 pistoles	make one	doubloon

Exchange from 38 *d.* to 42 *d.* sterling *per* piso.

EXAMPLES.

1. In 9764 *rials of plate*, how many pounds sterling, exchange at $41\frac{7}{8}$ *d. per piso?*

8)9764 *rials plate*

1220 - 10

d.				
40 *is*	$\frac{1}{6}$	203 -	8 -	4
1 *is*	$\frac{1}{40}$	5 -	1 -	$8\frac{1}{2}$
$\frac{4}{8}$ *is*	$\frac{1}{2}$	2 -	10 -	$10\frac{1}{4}$
$\frac{2}{8}$ *is*	$\frac{1}{2}$	1 -	5 -	5
$\frac{1}{8}$ *is*	$\frac{1}{2}$	-	12 -	$8\frac{1}{2}$

212 *l.* - 19 *s.* - $0\frac{1}{4}$ *the answer.*

2. In 8756 *rials vellon*, how many rials of plate? *Ans.* 4651 *rials plate*, 10 *qu.*

3. In 4651 *rials of plate*, 10 *q.* how many rials vellon? *Ans.* 8756

4. In 89641 *quartas*, how many pounds sterling, exchange at $39\frac{1}{2}$ *d. per* piastre? *Ans.* 115 *l.* 5 *s.* $2\frac{3}{4}$ *d.*

5. Reduce 7869 *rials vellon*, 19 *mar.* into pounds sterling, exchange $41\frac{1}{2}$ *d.* sterling *per piso.* *Ans.* 90 *l.* 7 *s.* $3\frac{1}{4}$ *d.*

6. In 89 *l.* 2 *s.* $11\frac{1}{2}$ *d.* sterling, how many rials of plate, &c. exchange at $40\frac{1}{8}$ *d. per* piece of eight? *Ans.* 4265 *rials plate*, 12 *q.*

7. In

7. In 2561 *pisos*, 5 *rials plate*, 3 *q.* how many pounds sterling, exchange $41\frac{1}{2}$ *d.* *Ans.* 442 *l.* 19 *s.* $0\frac{1}{4}$ *d.*

8. Bought goods in Spain to the value of 547268 *quartas*, exchange $40\frac{7}{8}$ *d.* sterling, how many pounds sterling must I sell them for in England to gain 20 *per cent.*? *Ans.* 873 *l.* 16 *s.* $2\frac{1}{2}$ *d.*

PORTUGAL.

Accounts are kept in Portugal in reas and milreas, and the exchange is by the milrea.

400 reas	make one	crusadoe
1000 reas, or $2\frac{1}{2}$ crusadoes	make one	milrea

Exchange from 60 *d.* to 67 *d. per* milrea.

EXAMPLES.

1. In 669 *mil.* 72 *reas*, how many pounds sterling, exchange 5 *s.* 7 *d.*?

s.	669 *milreas.*		
5 *is* $\frac{1}{4}$	167	5	
6 *is* $\frac{1}{10}$	16	14	6
1 *is* $\frac{1}{6}$	2	15	9
72 *reas* =	-	-	$4\frac{3}{4}$
	186 *l.*	15 *s.*	$7\frac{3}{4}$ *d.* *the answer.*

2. In 569 *l.* 17 *s.* 10 *d.* sterling, how many milreas, exchange at 5 *s.* 6 *d.* sterling *per* milrea? *Ans.* 2072 *milreas*, 333 *reas*

3. In 754 *l.* 18 *s.* 6 *d.* sterling, how many crusadoes, exchange $64\frac{1}{2}$ *d.*? *Ans.* $7022\frac{1}{2}$ *cru.*

4. In 2729 *crusadoes*, 372 *reas*, how much sterling, exchange at 62 *d.*? *Ans.* 282 *l.* 1 *s.* 10 *d.*

VENICE.

They keep their accounts at Leghorn in dollars, soldi and denari, and exchange by the ducat and piastre.

12 denari	make one	soldo
20 soldi	make one	lira, or piastre of Leghorn
$5\frac{1}{6}$ soldi	make one	grosso
24 grossi	make one	ducat.

Exchange from 52 *d.* to 54 *d. per* ducat, and from 45 *d.* to 54 *d. per* piastre.

Agio 20 *per cent.*

EXAMPLES.

1. In 7456 *pias.* 9 *sol.* 6 *den.* lire money, how many pounds sterling, exchange being at $49\frac{7}{8}$ *d. per* piastre?

d.		*pias.*	*s.*	*d.*
		7456	9	6
40	*is* $\frac{1}{6}$	1242	14	11
8	*is* $\frac{1}{5}$	248	10	$11\frac{3}{4}$
1	*is* $\frac{1}{8}$	31	1	$4\frac{1}{4}$
$\frac{4}{8}$	*is* $\frac{1}{2}$	15	10	8
$\frac{2}{8}$	*is* $\frac{1}{2}$	7	15	4
$\frac{1}{8}$	*is* $\frac{1}{2}$	3	17	8
		1549 *l.*	10 *s.*	11 *d.* *the answer.*

2. In 278 *l.* 17 *s.* 9 *d.* sterling, how many piastres of Leghorn, exchange at $47\frac{3}{8}$ *d. per* piastre?

Ans. 1412 *pias.* 16 *sol.* 8 *den.*

3. Reduce 1549 *duc.* 18 *sol.* 1 *den.* bank money of Venice, into sterling money, exchange at $47\frac{3}{4}$ *d.* sterling *per* ducat.

Ans. 290 *l.* 6 *s.* $2\frac{1}{4}$ *d.*

4. In 4789 *duc.* 19 *sol.* 3 *den.* current money, how many pounds sterling, exchange at 4 *s.* 1 *d. per* ducat banco, and agio 20 *per cent.*? *Ans.* 814 *l.* 16 *s.* 5 *d.*

5. In 415 *l.* 17 *s.* 4 *d.* sterling, how many ducats, &c. current, agio 20 *per cent.* and exchange at 53 *d. per* ducat?

Ans. 2259 *duc.* 19 *grossi*

6. In 100 *l.* sterling, how many piastres of Leghorn, exchange $52\frac{1}{2}$ *d. per* ducat? *Ans.* 2834 *pias.* 5 *sol.* 8 *den.*

RUSSIA.

They keep their accounts at Petersburg in rubles and copecs, and exchange by the ruble.

3 copecs	make one	altine
10 copecs	make one	grivena
25 copecs	make one	polpolitin
2 polpolitins	make one	politin
2 politins	make one	ruble
2 rubles	make one	ducat.

Russia exchanges with London by way of Hambro or Amsterdam, at the rate of 48 to 50 stivers *per* ruble; and sometimes directly to London from 4 *s.* to 5 *s. per* ruble.

EXAM-

EXAMPLES.

1. In 2634 *rub.* 58 *cop.* how many pounds sterling, exchange at 4*s.* 8*d.* sterling *per* ruble?

s.		2634 *rub.*	
4 *is* $\frac{1}{5}$		526	- 16
6 *is* $\frac{1}{8}$		65	- 17
2 *is* $\frac{1}{3}$		21	- 19
58 *cop.* =			2 - $8\frac{1}{4}$
		614*l.* - 14*s.* - $8\frac{1}{4}$*d.* *the answer.*	

2. In 674*l.* 17*s.* 6*d.* sterling, how many rubles, exchange 47 stivers *per* ruble, and 33*s.* $9\frac{1}{2}$ *d.* flemish *per* pound sterling? *Ans.* 2792 *rub.* 4 *gr.* 6 *cop.*
3. A merchant at London remits to his correspondent at Petersburg 471 *l.* 17*s.* 4*d.* *ster.* exchange 34*s.* 9*d.* flemish *per* pound *ster.* for Amsterdam, and the exchange from thence at 50 stivers *per* ruble, how many rubles must the correspondent receive? *Ans.* 1967 *rub.* 68 *cop.*
4. Received from Archangel *per* bill of exchange 4675 *rub.* 46 *cop.* exchange 122 copecs *per* rix-dollar of 50 stivers, and 34*s.* 7*d.* flemish *per* pound sterling: how much sterling is the sum? *Ans.* 923*l.* 9*s.* $1\frac{1}{4}$*d.*
5. In 4675 *rub.* 46 *cop.* how many pounds sterling? exchange 122 copecs *per* rix-dollar current, agio three *per cent.* and 34*s.* 7*d.* flemish *per* pound sterling. *Ans.* 896*l.* 11*s.* $2\frac{1}{4}$*d.*

POLAND AND PRUSSIA.

They keep their accounts at Dantzig in florins, gros, and penins, and exchange by the gros.

18 penins	make one	gros
18 gros	make one	oort
30 gros	make one	florin or polish guilder
3 florins	make one	rix-dollar
2 rix-dollars	make one	gold ducat

Exchange is made with Poland and Prussia by way of Holland, the exchange being from 240 to 295 grossi *per* pound flemish.

EXAMPLES.

1. In 478*l.* 14*s.* 9*d.* sterling, how many Prussia florins, &c. exchange 255 grossi *per* pound flemish, and 33*s.* 6*d.* flemish *per* pound sterling?

		478 *l.*	14 *s.*	9 *d.*
10 *s.*	*is* $\frac{1}{2}$	239	- 7	- $4\frac{1}{2}$
3 *s.* 4 *d.*	*is* $\frac{1}{6}$	79	- 15	- $9\frac{1}{2}$
2 *d.*	*is* $\frac{1}{20}$	3	- 19	- 9
		801 *l.*	- 17 *s.*	- 8 *d.*
				$8\frac{1}{2}$ = 255 *grossi.*
		6415	- 1	- 4
		400	- 18	- 10

6816 *florins, the answer.*

2. In 6949 *flor.* 14 *g.* 2 *pen.* Polish, how many pounds sterling, exchange, 260 $\frac{1}{2}$ Polish grossi, *per* pound flemish, and 34 *s.* 8 *d.* flemish *per* pound sterling? *Ans.* 461 *l.* 14 *s.* $5\frac{3}{4}$ *d.*

3. In 875 *l.* 14 *s.* 8 *d.* sterling, how many rix-dollars, &c. Polish, exchange 290 grossi Polish *per* pound flemish, and 34 *s.* 4 *d.* flemish *per* pound sterling? *Ans.* 4844 *rix-doll.* 9 *g.* 1 *pen.*

4. In 674 *l.* 18 *s.* 4 *d.* sterling, how many Polish guilders, &c. exchange 274 Polish grossi *per* pound flemish, and 35 *s.* 6 *d.* flemish *per* pound sterling? *Ans.* 10941 *guil.* 15 *g.* 12 *pen.*

5. In 546 *l.* 17 *s.* 8 *d.* sterling, how many gold ducats, exchange 295 grossi *per* pound flemish, and 33 *s.* 10 *d.* flemish *per* pound sterling? *Ans.* 1516 *duc.* 37 *g.* 7 *pen.*

S W E D E N.

They keep their accounts at Stockholm in copper dollars and oorts, or in silver dollars, and exchange by the copper dollar.

8 penins	make one	runstychen
3 runstychens		stiver, or whitton
8 stivers		marc
10 stivers and 2 runstychens, or 32 runstychens		copper dollar
3 copper dollars and 32 stiv. or 96 runstychens, or 4 marc.		silver dollar
24 marcs		copper rix-dollar.

The exchange here is subject to great variations, but is usually from 46 to 50 copper dollars *per* pound sterling.

EXAM-

EXAMPLES.

1. In 146*l.* 17*s.* 6*d.* ſterling, how many copper dollars, exchange 48 ½ copper dollars *per* pound ſterling?

146 *l.*	17 *s.*	6 *d.*	
		6	
881	5	-	-
		8	
7050	-	-	-
73	8	9	
7123	8	9	
		8	
	5)70	-	-
	14		

Anſ. 7123 *copp. doll.* 14 *run.*

2. In 546*l.* 19*s.* 6½*d.* ſterling, how many ſilver dollars, exchange 49 ½ copper dollars *per* pound ſterling?
Anſ. 9025 *ſil. doll.* 11 *run.* 5 *pen.*

3. In 674*l.* 11*s.* 6*d.* ſterling, how many marcs, &c. exchange 48 copper dollars *per* pound ſterling?
Anſ. 43172. *marcs,* 6 *ſt.* 9. *pen.*

4. In 11676 *ſilver doll.* 18 *run.* 7 *pen.* how many pounds ſterling, exchange 49 copper dollars *per* pound ſterling?
Anſ. 714*l.* 17*s.* 4½ *d.*

5. In 111*l.* 5*s.* 2½*d.* ſterling, how many Daniſh rix-dollars, exchange 35*s.* 7*d.* flemiſh *per* pound ſterling, 106 Amſterdam rix-dollars current for 100 Daniſh rix-dollars, and agio 3¾?
Anſ. 465 *dan. rix-doll.*

IRELAND.

Accounts are kept in Ireland in pounds, ſhillings and pence Iriſh, divided as in England; but having no coins of their own, they are ſupplied by the different countries with which they traffic.

The courſe of exchange between England and Ireland is from 5 to 12 *per cent*, according to the balance of trade.

EXAMPLES.

1. London remits to Ireland 787*l.* 15*s.* sterling; how much Irish must London be credited, exchange at $10\frac{1}{2}$ *per cent.*?

```
 787 l. 15 s.
        10
 ------------
 7877 - 10
        10
 ------------
78775 - —
  393 - 17 - 6
 ----------------
791.68 - 17 - 6
    20
 ------
 13.77
    12
 ------
  3.30
     4
 ------
  1.20
```

Ans. 791*l.* 13*s.* $3\frac{1}{4}$*d.*

2. Ireland remits to London 879*l.* 6*s.* 6*d.* Irish; how much sterling must Ireland be credited, exchange $11\frac{5}{8}$ *per cent.*?

Ans. 787*l.* 15*s.* *ster.*

3. London remits to Ireland, 540*l.* 10*s.* sterling; how much Irish must London be credited, exchange 12 *per cent.*?

Ans. 605*l.* 7*s.* 2*d.*

AMERICA, AND THE WEST INDIES.

In America and the West Indies, accounts are kept in pounds, shillings and pence as in England, which money is called currency.

The scarcity of cash obliges them to substitute bills for carrying on their trade; which being subject to many casualties, suffer a great discount in their negotiation.

EXAMPLES.

1. Philadelphia is indebted to London 1575*l.* 14*s.* 9*d.* currency; what sterling may London reckon to be remitted when the exchange is 35 *per cent.*?

175*l.*

1575 *l.* 14 *s.* 9 *d.*
4

6302 - 19 - -
5

3)31514 - 15 - -

9)10504 - 18 - 4

1167 *l.* 4 *s.* 3 *d.* *the anſwer.*

2. London conſigns to Virginia goods amounting to 578 *l.* 19 *s.* 6 *d.* which are ſold for 847 *l.* 15 *s.* 6 *d.* currency, what ſterling ought the factor to remit, deducting 5 *per cent.* for commiſſion and charges, and what does London gain *per cent.* upon the adventure, ſuppoſing the exchange at 30 *per cent.*? *Anſ.* 8 *l.* 9 *s.* 3 $\frac{1}{4}$ *d.*

3. Virginia is indebted to London 575 *l.* 19 *s.* 6 *d.* ſterling; with how much currency will London be credited at Virginia, when the exchange is 33 $\frac{1}{3}$ *per cent.*?

Anſ. 767 *l.* 19 *s.* 4 *d.*

ARBITRATION OF EXCHANGES.

As the price of exchange, in every place, is continually varying, the arbitration is nothing more than a method of finding ſuch a rate of exchange between any two places, as ſhall be in proportion with the rates aſſigned between each of them and a third place.

And it is by comparing the par of exchange, thus found, with the preſent courſe of exchange, that a perſon can judge which way to remit or draw to the moſt advantage, and what the advantage ſhall be.

All queſtions in this rule may be performed by one or more operations in the rule of three *.

EXAM-

* Any number of operations in the rule of three may be reduced into a ſingle one, thus:

Multiply the conſequents of all the proportions into one another continually for a dividend; and all the antecedents, except the firſt, for a diviſor; then will the quotient, ariſing from this diviſion, be the anſwer required.

Example. The exchange between London and Amſterdam is 1 *l.* ſterling for 38 *s.* flemiſh; betwixt Amſterdam and Francfort it is 6 *s.* flemiſh for

EXAMPLE.

1. If the exchange between London and Amsterdam be 33*s.* 9*d. per* pound sterling, and the exchange between London and Paris be 32 *d. per* ecu: what is the par of arbitration between Amsterdam and Paris?

$$240d. : 33s.\ 9d. :: 32$$

```
       12
      ----
      405
       32
      ----
      810
     1215
     -----
24,0)1296,0(54
     120
     ---
      96
      96
      --
```

Ans. 54 *d. flem. per ecu.*

2. Amsterdam changes on London at 34*s.* 4*d. per* pound sterling, and on Lisbon at 52 *d.* flemish for 400 reas: how ought the exchange to go between London and Lisbon?

Ans. 75 $\frac{75}{103}$ *sterling per milrea.*

3. London exchanges on Amsterdam at 34*s.* 9*d. per* pound sterling, and on Lisbon at 5*s.* 5$\frac{5}{8}$*d. per* milrea: what is the arbitrated price between Amsterdam and Lisbon?

Ans. 45 $\frac{39}{64}$ *flem. per crusadoe.*

4. London is indebted to Petersburg 5000 *rubles*: now the exchange between Petersburg and England is at 50*d. per* ruble; between Petersburg and Holland 90 *d. per* ruble;

---

for 65 cruitzers; and between Francfort and Paris it is 56 cruitzers for a crown: what is the exchange between London and Paris?

Lond.	*Amst.*	*Franc.*	*Paris.*
1 *l.* =	38 *s.*		
	6 *s.* =	66 *cru.*	
		54 *cru.* =	1 *cr.*

$$\frac{1 \times 38 \times 66}{6 \times 54 \times 1} = \frac{2508}{324} = 7\tfrac{20}{27} \text{ crowns, the answer.}$$

Compound arbitration of exchanges is only a continuation of several statings in simple arbitration,

and

and between Holland and England 36*s.* 4*d.*: which will be the most advantageous method for London to be drawn upon? *Ans. London will gain* 9*l.* 11*s.* $1\frac{3}{4}$*d. by making payment by way of Holland.*

5. Amsterdam has orders to remit a certain sum to Cadiz; at the time of this order Amsterdam can remit to Cadiz at $94\frac{3}{4}$*d.* *per* ducat of 375 marvadies, and London to Cadiz at 38*d.* *per* piastre of 272 marvadies: which will be the most advantageous for Amsterdam to remit directly to Cadiz, or by London, being 10 *guild.* 35 *st. per* pound sterling?

Ans. 18*s.* 8*d.* $\frac{1}{4}$ *per cent. in favour of Amsterdam.*

6. A merchant at London has 6000 guilders in the bank at Amsterdam, and was offered 22*d.* sterling apiece for them; but not liking the offer, he indorsed a bill for the whole to his factor at Paris; who brought the money to France, by exchanging at 55*d.* flemish *per* crown. He allowed the factor $\frac{1}{2}$ *per cent.* commission for his trouble, and then drew upon him for the whole, exchange at 32*d.* *per* ecu: how much was this better than the offer at 22*d.* *per* guilder?

Ans. 28*l.* 18*s.* 2*d.*

SOME OF THE MOST USEFUL PROPERTIES OF NUMBERS, EXTRACTED FROM EUCLID, AND OTHER WRITERS.

DEFINITIONS.

1. *Unity*, is that by which every thing in nature is called one.

2. *Number*, is that which is composed of one or more units.

3. *A multiple* of any number, is that which contains it some exact number of times.

4. One number is said to *measure* another, when it divides it without leaving any remainder.

5. And if a number exactly divides two, or more numbers, it is then called their *common measure.*

6. *An even number*, is that which can be halved, or divided into two equal parts.

7. *An odd number*, is that which cannot be halved, or which differs from an even number by unity.

8. *A Prime number*, is that which can only be measured by 1, or unity.

9. One number is said to be *prime* to another when unity is the only number by which they can both be measured.

10. A

12. *A Composite number,* is that which can be measured by some number greater than unity.

11. *A perfect number,* is that which is equal to the sum of all its aliquot parts.

Axiom 1. Any even number may be represented by 2 A, and any odd number by 2 A + 1.

2. The sum, difference, or product of any two whole numbers is a whole number.

PROPOSITIONS.

1. The sum of any number of even numbers is an even number.

For, let 2 A, 2 B, 2 C, *&c.* = *to any even numbers,*

Then will 2 A + 2 B + 2 C, *&c. be* = *to their sum.*

Which is, evidently, an even number, because it can be divided by 2. *(Def. 6.)*

2. The sum of any even number of odd numbers is an even number.

For, let 2 A + 1, 2 B + 1, 2 C + 1, 2 D + 1, *&c.* = *any odd numbers.*

Then will 2 A + 2 B + 2 C + 2 D, *&c.* + 1 + 1 + 1 + 1, *&c.* = *to their sum.*

And, since 2 A + 2 B + 2 C + 2 D, *&c. is an even number, and any even number of units is also an even number, it is plain that the whole must be even.*

3. The sum of any odd number of odd numbers, is an odd number.

For, let 2 A + 1, 2 B + 1, 2 C + 1, *&c.* = *any odd numbers.*

Then, 2 A + 2 B + 2 C, *&c.* + 1 + 1 + 1, *&c.* = *their sum.*

And, since 2 A + 2 B + 2 C, *&c.* = *to an even number, and any odd number of units is an odd number, the whole must be odd.*

4. If an even number be taken from an even number, the remainder will be even.

For, let 2 A *and* 2 B = *any two even numbers, of which* 2 A *is the greatest.*

Then, since 2 A — 2 B *is divisible by* 2, *it is evidently an even number.*

5. If an odd number be taken from an odd number the remainder will be even.

For, let 2 A + 1 *and* 2 B + 1 = *any two odd numbers, of which* 2 A + 1 *is the greatest.*

Then

Then, since $(2A+1)-(2B+1)$ *which is* $=2A-2B$ *is divisible by* 2, *it is evidently an even number.*

6. If an even number be taken from an odd one, or an odd number from an even one, the remainder will be odd.

For, let $2A$, $2B=$ *two even numbers, and* $2C+1$, $2D+1=$ *two odd ones, of which* $2C+1$ *is greater than* $2A$, *and* $2B$ *greater than* $2D+1$.

Then, since $2C+1-2A$ *(or* $2C-2A+1$*) and* $2B-2D-1$ *are not divisible by* 2, *they will evidently be odd numbers.*

7. If an odd number be multiplied by an odd number the product will be odd.

For, let $2A+1$ *and* $2B+1=$ *any two odd numbers.*

Then, will $4AB+2B+2A+1=$ *to their product, which is evidently an odd numbers because it is not divisible by* 2.

8. If an even number be multiplied by any number, either even or odd, the product will be even.

For, let $2A$, $2B$, *be any even numbers, and* $2C+1$ *an odd one.*

Then, will their products $2A\times(2C+1)$ *and* $2B\times(2C+1)$ *be, evidently even numbers, being divisible by* 2.

9. If an odd number measures an odd number, the quotient will be odd.

For, let $(A+1)\div(B+1)$ *or* $\frac{A+1}{B+1}=Q$; *then will* $(B+1)\times Q=A+1$:

And, because $B+1$ *and* $A+1$ *are odd numbers,* Q *must also be an odd number (Prop.* 7.)

10. If an odd, or even number measures an even one, the quotient will be even.

For, let $2A\div(2B+1)$, *or* $\frac{2A}{2B+1}=Q$; *then* $(2B+1)\times Q=2A$:

And, because $2B+1$ *is an odd aumber, and* $2A$ *is an even one,* Q *also must be an even one. (Prop.* 8.)

Again, let $2A\div 2B$, *or* $\frac{2A}{2B}=Q$; *then* $2B\times Q=2A$;

And, since $2A$ *and* $2B$ *are even numbers,* Q *must likewise be an even number. (Prop.* 8.)

Coroll. It appears also from Prop. 8. that an even number cannot measure an odd one.

11. If

11. If an odd or even number measures an even one, it will also measure the half of it.

For, let $2A \div (2B+1)$, *or* $\frac{2A}{2B+1} = Q$; *then* $\frac{A}{2B+1} = \frac{1}{2}Q$

But, Q *is an even number (Prop.* 10); *therefore* $\frac{A}{2B+1}$ *or its equal* $\frac{1}{2}Q$ *must be an whole number.*

Again, let $2A \div 2B$, *or* $\frac{2A}{2B} = Q$; *then* $\frac{A}{2B} = \frac{1}{2}Q$:

But since Q *is an even number (Prop.* 10); $\frac{1}{2}Q$ *must be a whole number.*

12. If one number measures another, it will also measure any multiple of it.

For, let n = *any number whatever, and* $A \div B$, *or* $\frac{A}{B} = Q$; *then* $\frac{nA}{B} = nQ$:

But since Q *is a whole number (by hyp.),* nQ *must, also, be an whole number (Ax.* 2).

13. If a number measures the whole of any number, and a part of it, it will also measure the remainder.

For, since $\frac{A+B}{C}$, *and* $\frac{A}{C}$ *are each of them whole numbers (by hyp.)*

$\frac{A+B}{C} - \frac{A}{C} = \frac{B}{C}$ *is also a whole number (Ax.* 2).

14. If a number measures two other numbers, it will also measure their sum and difference.

For, since $\frac{A}{C}$ *and* $\frac{B}{C}$ *are each of them whole numbers (by hyp.)*

$\frac{A+B}{C}$ *and* $\frac{A-B}{C}$ *must be also whole numbers (Ax.* 2).

15. The sum or difference of two numbers will measure the difference of their squares.

For $(A^2 - B^2) \div (A - B)$, *or* $\frac{A^2 - B^2}{A - B} = A + B$

And $(A^2 - B^2) \div (A + B)$, *or* $\frac{A^2 - B^2}{A + B} = A - B$

16. The

16. The ſum of two numbers will meaſure the ſum of their cubes; and the difference of two numbers will meaſure the difference of their cubes.

For, $(A^3+B^3) \div (A+B)$, *or* $\frac{A^3+B^3}{A+B} = A^2 - AB + B^2$

And, $(A^3-B^3) \div (A-B)$, *or* $\frac{A^3-B^3}{A-B} = A^2 + AB + B^2$

17. If a ſquare meaſures a ſquare, or a cube a cube, the root will alſo meaſure the root.

For, ſince $\frac{A^2}{B^2}$ *and* $\frac{A^3}{B^3}$ *are each of them whole numbers (by hyp.)*

$\frac{A}{B}$ *muſt alſo be a whole number, or otherwiſe whole numbers, multiplied by whole numbers, would not produce whole numbers.*

18. The product of two ſquare numbers is a ſquare number, and the product of two cube numbers, a cube number, &c.

Thus, $A^2 \times A^2 = A^4$, *whoſe ſquare root is* A^2.

And $A^3 \times A^3 = A^6$, *whoſe cube root is* A^3.

Cor. *Every power of a ſquare number is a ſquare, and every power of a cube number a cube.*

19. The ſum of two numbers, differing by unity, is equal to the difference of their ſquares.

Let A *and* $A+1$ *be the numbers,*

Then $2A+1 =$ *ſum; and* $(A+1)^2 - A^2 = A^2 + 2A + 1 - A^2 = 2A + 1 =$ *difference of their ſquares.*

Cor. *The differences of* $1^2, 2^2, 3^2, 4^2, 5^2$, *&c. are the odd numbers* 1, 3, 5, 7, 9, *&c.*

20. If an odd number (A) be prime to any other number (B), it will alſo be prime to the double of it ($2B$).

For no even number can meaſure A *(Cor. Prop.* 10*); and any odd number that meaſures* A *and* $2B$, *will alſo meaſure* A *and* B *(Prop.* 11*), in which caſe* A *and* B *would be prime to each other, which is abſurd.*

21. If two numbers, (A, B) be, each of them, prime to a third (C), their product (AB) will alſo be prime to it.

For, A *and* C *have no common factor, becauſe they are primes, nor* B *and* C, *for the ſame reaſon.*

Therefore AB *and* C *can have no common factor, whence they are primes.*

22. If one number (A) be prime to another (B), its square (A^2), cube (A^3), &c. will also be prime to it.

For, since A *and* B *have no common factor,*
Neither $A \times A$ (A^2) *nor* B *can have a common factor;*
Consequently they must be primes; and the same for any other power.

23. If two numbers (A and B) be prime to each other, their sum ($A + B$) will also be prime to either of them.

For, if not, let D *be the common measure of* A *and* $A + B$; *then it will also measure the remainder* B; *whence* A *would not be prime to* B: *which is contrary to the hypothesis.*

Cor. If a number ($A + B$) *be prime to one of its parts* (A), *it will also be prime to the remaining part* (B).

24. If any series of numbers, beginning from 1, be in continued geometrical proportion, the 3d. 5th. 7th. &c. will be squares; the 4th. 7th. 10th. &c. cubes; and the 7th. will be both a square and a cube.

Thus, in the series $1, r, r^2, r^3, r^4, r^5, r^6, r^7, r^8, r^9$,
r^2, r^4, r^6, r^8, *are squares;* r^3, r^6, r^9, *&c. cubes,*
and r^6 *is both a square and a cube.*

25. If N be made to represent any of the natural numbers, 1, 2, 3, 4, 5, &c. then will $6N - 1$ and $6N + 1$ constitute a series which contains all the prime numbers above 3.

Thus, if $N = 1, 2, 3, 5, 7$, *&c. we shall have* 5, 7, 11, 13, 17, 19, 29, 31, 41, 43 = *prime numbers.*

It must be observed, however, that neither $6N - 1$ *nor* $6N + 1$ *are always prime numbers, nor has any general expression been yet devised for this purpose.*

26. All the powers of any number, ending in 5, will also end in 5; and if a number ends in 6, all its powers will end in 6.

For $5 \times 5 = 25$; *and* $6 \times 6 = 36$, *and so on.*

27. No number is a square that ends in 2, 3, 7, or 8.

This will appear plain, by squaring all the natural numbers to 10.

28. A cube number may end in any of the natural numbers, 1, 2, 3, 4, 5, 6, 7, 8, 9 or 0.

This will, likewise, appear by cubing those numbers.

Cor. There is no such thing as the exact square root of 2, 3, 5, 6, 7, 8, 10, *&c. nor the exact cube root of* 2, 3, 4, 5, 6, *&c.*

29. Any even square number is divisible by 4.

For, since the root must be even, let 2n *be that root; then* $4n^2$ *is the square of it, which is evidently divisible by* 4.

Prop. 30.

30. An odd square number, divided by 4, leaves a remainder of 1.

The root of an odd square number is odd, let, therefore, $2n + 1$ *be that root; then* $4n^2 + 4n + 1$, *being divided by* 4, *leaves* 1.

MISCELLANEOUS QUESTIONS.

1. What part of 3*d.* is a third part of 2*d.*? *Ans.* $\frac{2}{9}$
2. A has by him $1\frac{1}{2}$ *cwt.* of tea, the prime cost of which was 96*l.* Now, granting interest to be at 5 *per cent.* it is required to find how he must rate it *per lb.* to B, so that by taking his negotiable note, payable at 3 months, he may clear 20 guineas by the bargain? *Ans.* 14*s.* $1\frac{13}{56}$ *d.*
3. What annuity is sufficient to pay off 50 millions of pounds in 30 years at 4 *per cent.* compound interest? *Ans.* 2891505 *l.*
4. Sold a piece of cloth containing 1000 flemish ells for 850 guineas, and gained upon every yard $\frac{1}{8}$ of the prime cost of an English ell: what did the whole piece stand me in? *Ans.* 771 *l.* 17 *s* $10\frac{2}{37}$ *d.*
5. The hour and minute hand of a clock are exactly together at 12 o'clock; when are they next together? *Ans.* 1 *h.* $5\frac{5}{11}$ *min.*
6. There is an island 73 miles in circumference, and 3 footmen all start together to travel the same way about it; A goes 5 miles a day, B 8, and C 10; when will they all come together again? *Ans.* 73 *days.*
7. Sold goods for 60 guineas, and by so doing lost 17 *per cent.* whereas I ought, in dealing, to have cleared 20 *per cent.*: how much were they sold under their just value? *Ans.* 28 *l.* 1 *s.* $8\frac{20}{83}$ *d.*
8. If, by selling goods at 2*s.* 3*d. per lb.* I clear *cent. per cent.*; what do I clear *per cent.* by selling them for 9 guineas *per cwt.* *Ans.* 50 *per cent.*
9. Laid out in a lot of muslin 500*l.* but upon examination, 3 parts in 9 proved to be damaged, so that I could make but 5*s. per* yard of it, and by so doing find I lost 50*l.* at what rate *per* ell must I sell the undamaged part, so that I may clear 50*l.* by the whole? *Ans.* 11*s.* $7\frac{2}{7}$ *d.*
10. A young hare starts 40 yards before a greyhound, and is not perceived by him till she has been up 40 seconds; she scuds away at the rate of 10 miles an hour, and the dog, on view, makes after her at the rate of 18: how long

long will the courſe hold, and what ground will be run over, beginning with the out-ſetting of the dog?

Anſ. $60\frac{5}{22}$ *ſec. and* 530 *yards run.*

11. A traveller leaves Exeter at 8 o'clock on Monday morning, and walks towards London, at the rate of 3 miles an hour, without intermiſſion; another traveller ſets out from London at 4 o'clock the ſame evening, and walks for Exeter at the rate of 4 miles an hour conſtantly; now, ſuppoſing the diſtance between the two cities to be 130 miles, whereabouts on the road will they meet? *Anſ.* $69\frac{3}{7}$ *miles from Exeter.*

12 A reſervoir for water has two cocks to ſupply it; by the firſt alone it may be filled in 40 minutes, and by the ſecond in 50 minutes; it has likewiſe a diſcharging cock, by which it may, when full, be emptied in 25 minutes. Now, if theſe three cocks are all left open when the water comes in, in what time would the ciſtern be filled, ſuppoſing the influx and efflux of the water to be always alike? *Anſ.* 3 *h.* 20 *min.*

13. A man being aſked how many ſheep he had in his drove, ſaid if I had as many more, half as many more, and ſeven ſheep and a half, I ſhould have 20: how many ſheep had he? *Anſ.* 5

14. A perſon left 40*s.* to 4 poor widows, A, B, C and D; to A he left $\frac{1}{3}$, to B $\frac{1}{4}$, to C $\frac{1}{5}$, and to D $\frac{1}{6}$, deſiring the whole might be diſtributed accordingly: what is the proper ſhare of each? *Anſ.* A's *ſhare* 14*s.* $0\frac{16}{38}$*d.* B's 10*s.* $6\frac{12}{38}$*d.* C's 8*s.* $5\frac{12}{38}$*d.* D's 7*s.* $0\frac{8}{38}$*d.*

15. How many oaken planks will floor a barn $60\frac{1}{2}$ feet long, and $33\frac{1}{2}$ wide; when the planks are 15 feet long, and 15 inches wide? *Anſ.* 108

16. The amount of a ſum of money which had been put out to intereſt is 100*l.* and the principal is juſt 7 times as much as the intereſt; what is the principal? *Anſ.* 87*l.* 10*s.*

17. What number is that of which 9 is $\frac{2}{3}$ of it? *Anſ.* $13\frac{1}{2}$

18. A perſon dying worth 5460*l.* left his wife with child, to whom he bequeathed, if ſhe had a ſon, $\frac{1}{3}$ of his eſtate, and the reſt to his ſon; but if ſhe had a daughter, $\frac{1}{3}$ to the daughter, and the reſt to her mother: Now it happened that ſhe had both a ſon and a daughter; how muſt the eſtate be divided to anſwer the father's intention? *Anſ. The daughter's part is* 780*l. the ſon's* 3120*l. and the mother's* 1560*l.*

19. A general diſpoſing of his army into a ſquare battle, finds he has 284 ſoldiers over and above; but increaſing each ſide

ſide with one ſoldier, he wants 25 to fill up the ſquare; how many ſoldiers had he? *Anſ.* 24000

20. I would put 60 hogſheads of London beer into 30 wine pipes, and deſire to know what the caſk muſt hold that receives the difference; 231 ſolid inches being the gallon of wine, and 282 that of beer?

Anſ. 143 *gal.* 2 *qu.* 32 *rem.*

21. A tradeſman increaſed his eſtate annually $\frac{1}{3}$ part, abating 100*l.* which he uſually ſpent in his family; and at the end of $3\frac{1}{4}$ years, found that his net eſtate amounted to 3179*l.* 11*s.* 8*d.* what had he at his outſetting?

Anſ. 1421*l.* 7*s.* $6\frac{1}{2}$*d.*

22. A perſon after ſpending $\frac{1}{3}$ of his yearly income plus 10*l.* had then remaining $\frac{1}{2}$ plus 15*l.*: what was his income?

Anſ. 150*l.*

23. There is a prize of 212*l.* 14*s.* 7*d.* to be divided amongſt a captain, 4 men, and a boy: the captain is to have a ſhare and a half; the men each a ſhare, and the boy $\frac{1}{3}$ of a ſhare: what ought each perſon to have? *Anſ. The captain* 54*l.* 14*s.* $\frac{2}{7}$*d. each man* 36*l.* 9*s.* $4\frac{2}{7}$*d. and the boy* 12*l.* 3*s.* $1\frac{3}{4}$*d.*

24. A ciſtern containing 60 gallons of water has 3 unequal cocks for diſcharging it; the greateſt cock will empy it in 1 hour; the ſecond in 2 hours, and the third in 3: in what time will it be empty if they all run together?

Anſ. $32\frac{8}{11}$ *minutes*

25. In an orchard of fruit trees $\frac{1}{2}$ of them bear apples, $\frac{1}{4}$ pears, $\frac{1}{6}$ plums, and 50 of them cherries: how many trees are there in all? *Anſ.* 600

26. A perſon who was poſſeſſed of a $\frac{3}{5}$ ſhare of a coppermine, ſold $\frac{3}{4}$ of his intereſt therein for 1710*l.*: what was the reputed value of the whole at the ſame rate?

Anſ. 3800*l.*

27. Suppoſe the ſea allowance for the common men to be 5 *lb's* of beef, and 3 *lb's* of biſcuit a day, for a meſs of 4 people; and that the price of the firſt is $2\frac{1}{4}$*d. per lb.* and of the ſecond $1\frac{1}{2}$*d.*; now, if the ſhip's company be ſuch that the meat they eat coſt the government 12 guineas *per* day; what muſt they pay for their bread *per* week?

Anſ. 35*l.* 5*s.* $5\frac{1}{2}$*d.*

28. If the ſcavenger's rate, at $1\frac{1}{2}$*d.* in the pound comes to 6*s.* $7\frac{1}{2}$*d.* where they uſually aſſeſs $\frac{4}{5}$ of the rent: what will the

the king's tax for that houſe be at 4*s.* in the pound, rated at the full rent? *Anſ.* 13*l.* 5*s.*

29. A can do a piece of work alone in 10 days, and B in 13; ſet them both about it together, in what time will it be finiſhed? *Anſ.* 5 $\frac{15}{23}$ *days*

30. B and C together can build a boat in 18 days: with the aſſiſtance of A they can do it in 11 days; in what time would A do it by himſelf? *Anſ.* 28 $\frac{2}{7}$ *days*

31. If A can do a piece of work alone in 10 days, and A and B together in 7 days; in what time can B do it alone? *Anſ.* 23 $\frac{1}{3}$ *days*

32. A, B and C can complete a piece of work together in 12 days; C can do it alone in 24 days, and A in 34 days; in what time could B do it by himſelf? *Anſ.* 81 $\frac{3}{5}$ *days*

33. A can do a piece of work in 3 weeks; B can do thrice as much in 8 weeks, and C 5 times as much in 12 weeks: in what time can they finiſh it jointly? *Anſ.* 5 *days*, 4 *hours*

34. If a cardinal can pray a fool out of purgatory, by himſelf, in an hour, a biſhop in 3 hours, a prieſt in five, and a friar in 7; in what time can they pray out 3 fools, all praying together? *Anſ.* 1 *ho.* 47 *m.* 23 $\frac{2}{11}$ *ſec.*

35. Bought 120 oranges at 2 a penny, and 120 more at 3 a penny, and ſold them altogether at 5 for 2*d.*: what did I gain or loſe by the bargain? *Anſ. Loſt* 4*d.*

36. A water tub holds 147 gallons; the pipe uſually brings in 14 gallons in 9 minutes; the tap diſcharges, at a medium, 40 gallons in 31 minutes; now, ſuppoſing theſe both to be careleſsly left open, and the water to be turned on at 2 o'clock in the morning; a ſervant at 5, finding the water running, ſhuts the tap, and is ſolicitous to know in what time the tub will be filled after this accident, in caſe the water continues to flow from the main.

Anſ. The tub will be full at 3 *min.* 48 $\frac{21}{31}$ *ſec. after* 6.

37. Part 1500*l.*; give B 72*l.* more than A, and C 112*l.* more than B. *Anſ.* A's *ſhare is* 414$\frac{2}{3}$*l.* B's 486$\frac{2}{3}$*l.* C's 598$\frac{2}{3}$*l.*

38. A and B venturing equal ſums of money clear by joint trade 154*l.*; by agreement A was to have 8 *per cent.* becauſe he ſpent his time in the execution of the project; and B was only to have 5: what was A allowed for his trouble? *Anſ.* 35*l.* 10*s.* 9$\frac{3}{13}$*d.*

39. A, B and C are to ſhare 100,000*l.* in the proportion of $\frac{1}{3}$, $\frac{1}{4}$ and $\frac{1}{5}$ reſpectively; but C's part being loſt by his death,

death, it is required to divide the whole sum properly between the other two.

Ans. A's *part is* $57142\frac{282}{329}$, *and* B's $42857\frac{47}{329}$.

40. A stationer sold quills at 11*s*. a thousand, by which he cleared $\frac{3}{8}$ of the money; but growing scarce he raised them to 13*s*. 6*d*. a thousand; what did he clear *per cent.* by the latter price? *Ans.* 96*l*. 7*s*. $3\frac{3}{11}$*d*.

41. Required the least number that can be divided by 1, 2, 3, 4, 5, 6, 7, 8 and 9 without leaving a remainder?

Ans. 2520

43. Suppose a man has a calf, which at the end of three years begins to breed, and afterwards brings a female calf every year; and that each calf begins to breed in like manner at the end of three years, bringing forth a cow calf every year; and that these last breed in the same manner, &c.; to determine the owner's whole stock at the end of 20 years? *Ans.* 1278

FINIS.

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